University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 310: Digital Signal Processing I
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Problem Set 1
Summer 2011
Reading: Chapter 1, Chapter 2, Appendices A and D

Problem 1
Evaluate and represent your final answer in both Cartesian and Polar forms. Try to simplify as far as you can by hand, without using a calculator or computer.

1. $(3\angle 150^\circ) + (5\angle -60^\circ) + (4\angle 120^\circ)$
2. $\frac{(-1+j)^5}{1+j}$
3. $\frac{5\angle 60^\circ}{2j} + \frac{\sqrt{2}e^{j\pi}}{2-j}$
4. $\left(\frac{-1+j3}{1-j} + \frac{3+j}{1-j2}\right)^n$

Problem 2
Sketch the following functions ($u[n]$ is the step function in the discrete time domain):

1. $n(u[n] - u[n-5])$
2. $u[n+2]u[n-3]$
3. $u[n-1] + nu[n-4]$

Problem 3
Compute the following:

1. Determine the roots of the equation $2z^3 + 1 = 0$.
2. Use the roots to factor the polynomial $G(z) = 2z^3 + 1$ as a product of first order polynomials in z.
3. Express $G(z)$ as a product of first and second order factors with *real coefficients*.

Problem 4
Evaluate the following integrals:

1. $\int_{-\infty}^{\infty} (t^2 + 5t + 6)\delta(t)dt$
2. $\int_{-\infty}^{-3} (t^2 + 5t + 6)\delta(t)dt$

3. $\int_{-3}^{\infty}(t^2+5t+6)\delta(t)dt$

4. $\int_{-\infty}^{\infty}(t^2+5t+6)\delta(t-3)dt$

5. $\int_{-\infty}^{\infty}(t^2+5t+6)\delta(3t-2)dt$

6. $\left[e^{-t}u(t)\right] * \delta(3t-2)$, where $*$ is convolution

Problem 5

Determine the Fourier transforms. Please note that $u(t)$ denotes the unit step in continuous time. [*hint: do not explicitly solve for* $X(\omega)$*; make use of the continuous time Fourier transform properties*]

1. $\delta(2t-3)$
2. $e^{-2\alpha t}u(t)$
3. $u(t)-u(t-T)$
4. $sin(2\Omega_0 t+\phi)$

Problem 6

Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental (smallest) period.

1. $x[n] = cos\left(\frac{\pi n}{3}\right)$
2. $x[n] = cos\left(\frac{3\pi n}{11}+1\right)$
3. $x[n] = sin\left(\frac{4\pi n}{3}\right)$
4. $x[n] = e^{\left(\frac{j\pi n}{4}\right)}$
5. $x[n] = sin\left(\frac{3n}{4}\right)$
6. $x[n] = sin\left(2\pi n\right)$
7. $x[n] = e^{j\pi n}$

Problem 7

The continuous-time Fourier transform is given by the following relation:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt.$$

For the signal, $x(t) = e^{-4t}(u(t)-u(t-2))$, determine the continuous-time Fourier transform of each of the following signals.

1. $x(t)e^{j5t}$
2. $x(t)+x(t+2)$
3. $x(t)cos(2t)$

4. $tx(t)$

Problem 8

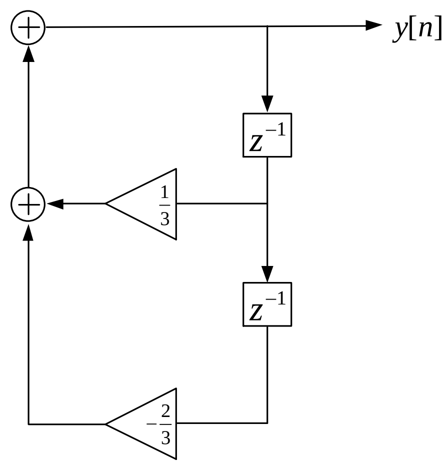
Suppose that $g(t) = x(t) * \frac{1}{\pi} sinc(t)$ (where $*$ denotes convolution) and that the Fourier transform is given by

$$G(\omega) = \begin{cases} 1 - |\omega|, & |\omega| \leq 1 \\ 0, & \text{else.} \end{cases}$$

1. Sketch $|G(\omega)|$ and $\angle G(\omega)$ each on their own plot over the region $|\omega| < 2$.
2. Determine $X(\omega)$ and then the corresponding x(t). Assume that $X(\omega) = 0$ for $|\omega| > 1$.
3. Determine $\int_{-\infty}^{\infty} |g(t)|^2 dt$.

Problem 9

Consider the following discrete-time system. If $y[-1] = 1$ and $y[-2] = -1$, write a difference equation for $y[n]$.



Problem 10

The discrete-time Fourier transform (DTFT) of a sequence $x[n]$ is given by the following relation:

$$X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}.$$

For each of the following sequences, determine the corresponding DTFT. [*hint: do not solve any of the summations*]

1. $x[n] = u[n+3] - u[n-5]$
2. $x[n] = 2\delta[n+3] + 2\delta[n-3]$
3. $x[n] = \left(\frac{1}{3}\right)^{n-4} u[n-4]$
4. $x[n] = (1-n)2^n u[n]$
5. Sketch the magnitude and phase for $\omega \in [-\pi, \pi]$ for parts (a) and (b).

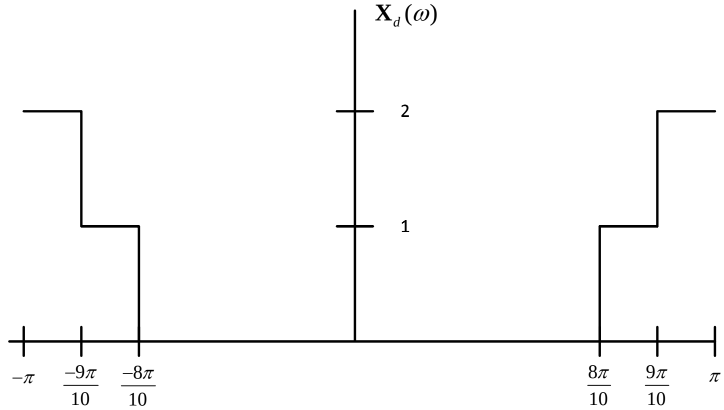
Problem 11

The DTFT of $x[n]$ is given for $\omega \in [-\pi, \pi]$. Determine the signal $x[n]$ corresponding to each of the following:

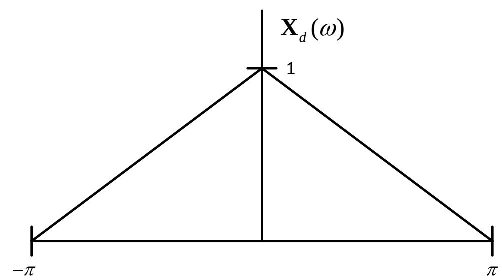
1. $X_d(\omega) = 2 + 3e^{-j\omega} + 2e^{-3j\omega} - e^{-5j\omega}$

2. $X_d(\omega) = \begin{cases} 0, & 0 \leq |\omega| \leq \omega_0 \\ 1, & \omega_0 \leq |\omega| \leq \pi \end{cases}$

3.



4. [*hint: make use of the following property,* $\mathcal{F}^{-1}\{X_1(\omega) \star X_2(\omega)\} = x_1[n]x_2[n]$]



Problem 12

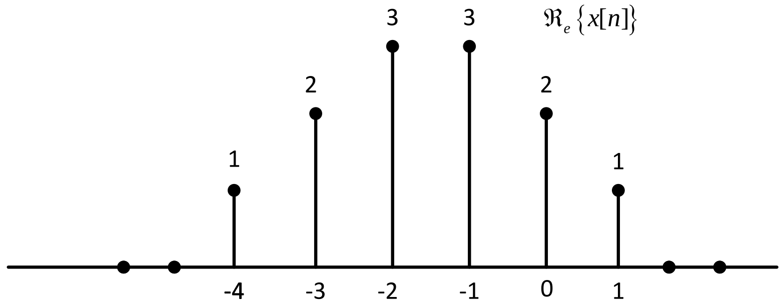
Let $X_d(\omega)$ denote the DTFT of the complex valued signal $x[n]$, where the real and imaginary parts of $x[n]$ are given below. [*hint: you can perform the following calculations without explicitly evaluating* $X_d(\omega)$]

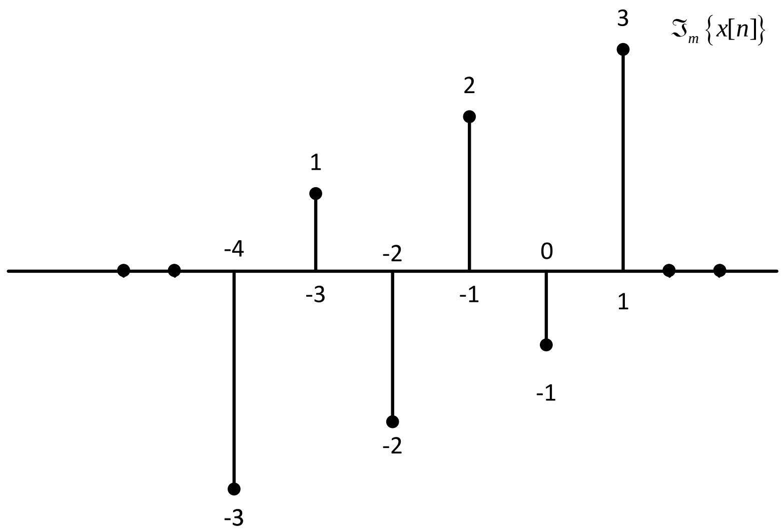
1. Evaluate $X_d(\omega)|_{\omega=0}$

2. Evaluate $X_d(\omega)|_{\omega=\pi}$

3. Evaluate $\int_{-\pi}^{\pi} X_d(\omega)d\omega$

4. Determine and sketch the signal whose DTFT is $X_d^*(-\omega)$





Problem 13
The discrete Fourier transform (DFT) of a finite-length sequence $x[n]$, defined only over the range $0 \leq n \leq N-1$, is given by

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}}, \quad 0 \leq k \leq N-1$$

For each of the following finite length sequences, determine the corresponding DFT, $X[k]$.

1. $x[n] = \delta[n-3], 0 \leq n \leq 3$
2. $x[n] = \begin{cases} 1 & 0 \leq n \leq 2 \\ 0 & 3 \leq n \leq 5 \end{cases}$
3. $x[n] = cos\left(\frac{\pi n}{4}\right), 0 \leq n \leq 7$
4. $x[n] = \begin{cases} 1 & n \text{ even}, & 0 \leq n \leq 6 \\ 0 & n \text{ odd}, & 0 \leq n \leq 6 \end{cases}$
5. Sketch the magnitude and phase for parts (a) and (b).

***Reminder - Homework is due on 06/24/2011 at 3:00 PM - place your assignments in the ECE 410 homework drop box in Everitt Hall!**