University of Illinois at Urbana-Champaign

ECE 310: Digital Signal Processing

PROBLEM SET 1: SOLUTIONS

Chandra Radhakrishnan *Peter Kairouz*

Problem 1

Solution:

1. $(3\angle 150^\circ) + (5\angle -60^\circ) + (4\angle 120^\circ)$

$$\begin{aligned}
&= 3cos(150^\circ) + 5cos(-60^\circ) + 4cos(120^\circ) + j(3sin(150^\circ) + 5sin(-60^\circ) + 4sin(120^\circ)) \\
&= 3\left(\frac{-\sqrt{3}}{2}\right) + 5\left(\frac{1}{2}\right) + 4\left(\frac{-1}{2}\right) + j\left(3\left(\frac{1}{2}\right) + 5\left(\frac{-\sqrt{3}}{2}\right) + 4\left(\frac{-\sqrt{3}}{2}\right)\right) \\
&= \frac{-3\sqrt{3}+1}{2} + j\left(\frac{3-\sqrt{3}}{2}\right) \\
&= 2.1918e^{j2.8481}
\end{aligned}$$

2. $\frac{(-1+j)^5}{1+j}$

$$= \frac{(\sqrt{2})^5 e^{(\frac{j3\pi}{4})^5}}{\sqrt{2}e^{\frac{j\pi}{4}}} = \frac{4\sqrt{2}e^{\frac{j15\pi}{4}}}{e^{\frac{j\pi}{4}}} = 4e^{\frac{j6\pi}{4}} = 4e^{\frac{j3\pi}{2}} = -4j$$

3. $\frac{5\angle 60^\circ}{2j} + \frac{\sqrt{2}e^{j\pi}}{2-j}$

$$\begin{aligned}
&= \frac{5/2 + j5\sqrt{3}/2}{2j}\left(\frac{-2j}{-2j}\right) + \frac{\sqrt{2}(-1)}{2-j}\left(\frac{2+j}{2+j}\right) = \frac{-5j+5\sqrt{3}}{4} + \frac{-2\sqrt{2}-\sqrt{2}j}{5} \\
&= \frac{-2\sqrt{2}}{5} + \frac{5\sqrt{3}}{4} + j\left(\frac{-5}{4} - \frac{-\sqrt{2}}{5}\right) = 2.215e^{-j0.7642}
\end{aligned}$$

4. $\left(\frac{-1+j3}{1-j} + \frac{3+j}{1-j2}\right)^n$

$$= \left(\frac{(-1+j3)(1+j2)+(3+j)(1-j)}{(1-j)(1+j2)}\right)^n = (-1)^n = e^{j\pi n}$$

Problem 2

Sketch the following functions ($u[n]$ is the step function in the discrete time domain):

1. The function $n(u[n] - u[n-5])$ is shown in Fig. 1
2. The function $u[n+2]u[n-3]$ is sketched in Fig. 2
3. the function $u[n-1] + nu[n-4]$ is shown in Fig. 3

Problem 3

Assume $z = re^{j\theta}$

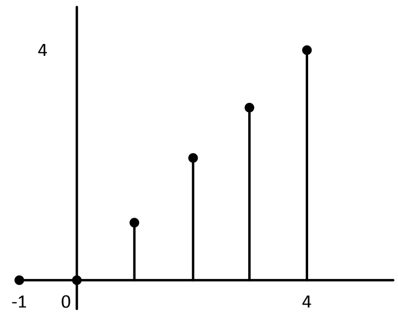


Figure 1: Problem 2. Figure for Problem 2 (1).

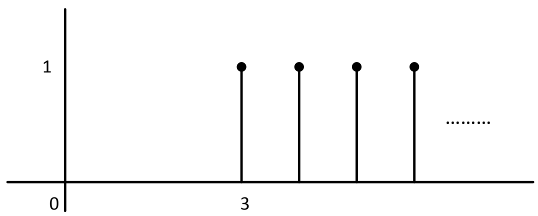


Figure 2: Problem 2. Figure for Problem 2 (2).

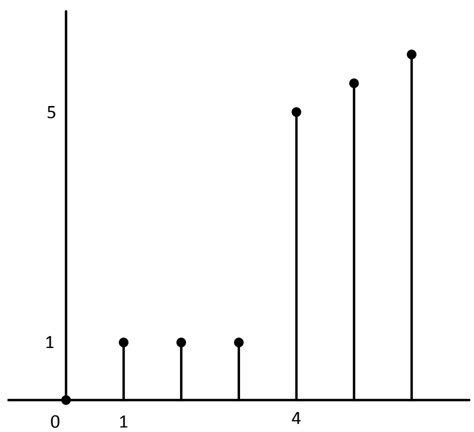


Figure 3: Problem 2. Figure for Problem 2 (c).

1. Consider $2z^3 + 1 = 0$. This can be written as,

$$\begin{aligned} 2r^3 e^{j3\theta} + 1 &= 0 \\ r^3 e^{j3\theta} &= \frac{-1}{2} \\ r &= \frac{1}{\sqrt[3]{2}} \\ e^{j3\theta} &= e^{\pi + 2k\pi} \\ \\ \theta &= \frac{\pi + 2k\pi}{3} \end{aligned}$$

The values of θ can be found as follows,

$$\begin{aligned} k=0 \quad &, \quad \theta=\frac{\pi}{3} \\ k=1 \quad &, \quad \theta=\pi \\ k=2 \quad &, \quad \theta=\frac{5\pi}{3}=\frac{-\pi}{3} \end{aligned}$$

After this the values of θ repeat. Hence we have,

$$z=\frac{1}{\sqrt[3]{2}}e^{j\frac{\pi}{3}}, \frac{1}{\sqrt[3]{2}}e^{j\pi}, \frac{1}{\sqrt[3]{2}}e^{j\frac{5\pi}{3}}$$

2. The polynomial $G(z)$ can be written as,

$$G(z)=\left(z+\frac{1}{\sqrt[3]{2}}\right)\left(z-\frac{1}{\sqrt[3]{2}}e^{j\frac{\pi}{3}}\right)\left(z-\frac{1}{\sqrt[3]{2}}e^{j\frac{5\pi}{3}}\right)$$

3. $G(z)$ can be written as,

$$\left(z+\frac{1}{\sqrt[3]{2}}\right)\left(z^2-\frac{z}{\sqrt[3]{2}}+\frac{1}{2^{\frac{2}{3}}}\right)$$

Problem 4

1. $\int\limits_{-\infty}^{\infty}(t^2+5t+6)\delta(t)dt=6$

2. $\int\limits_{-\infty}^{-3}(t^2+5t+6)\delta(t)dt=0$

3. $\int\limits_{-3}^{\infty}(t^2+5t+6)\delta(t)dt=6$

4. $\int\limits_{-\infty}^{\infty}(t^2+5t+6)\delta(t-3)dt=30$

5. $\int\limits_{-\infty}^{\infty}(t^2+5t+6)\delta(3t-2)dt$

$$\begin{aligned} &=\int\limits_{-\infty}^{\infty}\frac{1}{3}(t^2+5t+6)\delta(t-2/3)dt \\ &=\frac{1}{3}\left(\frac{4}{9}+5\times\frac{2}{3}+6\right) \\ &=3.259 \end{aligned}$$

6. $[e^{-t}u(t)]*\delta(3t-2)$, where $*$ is convolution

$$e^{-t}u(t)*\delta(3t-2)=\frac{1}{3}e^{(t-2/3)}u(t-2/3)$$

Problem 5

The facts used here are:

$$\int\limits_{-\infty}^{\infty}\delta(t)e^{-j\Omega t}=1$$

and,

$$\int_{-\infty}^{\infty} e^{-j\Omega t} = 2\pi\delta(\Omega)$$

1. $\delta(2t-3)$ Using Shifting property of Fourier Transform,

$$\begin{aligned} (t-t_0) &\leftrightarrow F(\Omega)e^{-j\Omega t_0} \\ \delta(2t-3) &\leftrightarrow \frac{1}{2}e^{-j\Omega 3/2} \end{aligned}$$

2. $e^{-2\alpha t}u(t)$

$$\begin{aligned} F\left(e^{-2\alpha u(t)}\right) &= \int_{-\infty}^{\infty} e^{-2\alpha t}u(t)e^{-j\Omega t}dt \\ &= \int_{0}^{\infty} e^{-2\alpha t}e^{-j\Omega t}dt \\ &= \int_{0}^{\infty} e^{t(-2\alpha+j\Omega)}dt \\ &= \frac{-1}{2\alpha+j\Omega}\left[e^{-t(2\alpha+j\Omega)}\right]_0^{\infty} \\ &= \frac{1}{2\alpha+j\Omega} \end{aligned}$$

3. $u(t)-u(t-T)$

$$\begin{aligned} F\left(u(t)-u(t-T)\right) &= \int_{-\infty}^{\infty}(u(t)-u(t-T))e^{-j\Omega t}dt \\ &= \int_{0}^{T} e^{-j\Omega t}dt \\ &= \frac{1}{j\Omega}(1-e^{-j\Omega T}) \\ &= Te^{\frac{-j\Omega T}{2}}\operatorname{sinc}\left(\frac{\Omega T}{2}\right) \end{aligned}$$

4. $sin(2\Omega_0 t+\phi)$

$$\begin{aligned} F\left(sin(2\Omega_0 t+\phi)\right) &= F\left[\frac{e^{j(2\Omega_0 t+\phi)}-e^{-j(2\Omega_0 t+\phi)}}{2j}\right] \\ &= \frac{1}{2j}\left[e^{j\phi}F[e^{j2\Omega_0 t}]-e^{-j\phi}F[e^{-j2\Omega_0 t}]\right] \\ &= -j\pi\left[e^{j\phi}\delta(\Omega-2\Omega_0)-e^{-j\phi}\delta(\Omega+2\Omega_0)\right] \\ &= j\pi\left[e^{-j\phi}\delta(\Omega+2\Omega_0)-e^{j\phi}\delta(\Omega-2\Omega_0)\right] \end{aligned}$$

Problem 6

Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental (smallest) period.

1. $x[n] = cos\left(\frac{\pi n}{3}\right)$

$$\begin{aligned}\frac{\pi}{3}(n+N) &= \frac{\pi}{3}n + 2\pi k\\ \frac{\pi}{3}N &= 2\pi k\\ \frac{N}{6} &= k\end{aligned}$$

   Hence periodic with period $N = 6$

2. $x[n] = cos\left(\frac{3\pi n}{11} + 1\right)$

$$\begin{aligned}\frac{3\pi(n+N)}{11} + 1 &= \frac{3\pi n}{11} + 1 + 2\pi k\\ \frac{3\pi N}{11} &= 2\pi k\\ \frac{3N}{22} &= k\end{aligned}$$

   Hence periodic with period $N = 22$

3. $x[n] = sin\left(\frac{4\pi n}{3}\right)$

$$\begin{aligned}\frac{4\pi(n+N)}{3} &= \frac{4\pi n}{3} + 2\pi k\\ \frac{4\pi N}{3} &= 2\pi k\\ \frac{2N}{3} &= k\end{aligned}$$

   Hence periodic with period $N = 3$

4. $x[n] = e^{\left(\frac{j\pi n}{4}\right)}$

$$\begin{aligned}\frac{\pi(n+N)}{4} &= \frac{\pi n}{4} + 2\pi k\\ \frac{\pi N}{4} &= 2\pi k\\ \frac{N}{8} &= k\end{aligned}$$

   Hence periodic with period $N = 8$

5. $x[n] = sin\left(\frac{3n}{4}\right)$

$$\frac{3(n+N)}{4} = \frac{3n}{4} + 2\pi k$$

   Non-periodic since it has no integer solution

6. $x[n] = sin\left(2\pi n\right)$

$$\begin{aligned}2\pi(n+N) &= 2\pi n + 2\pi k\\ 2\pi N &= 2\pi k\end{aligned}$$

   Hence periodic with period $N = 1$

7. $x[n] = e^{j\pi n}$

$$\begin{aligned} \pi(n+N) &= \pi n + 2\pi k \\ \pi N &= 2\pi k \end{aligned}$$

Hence periodic with period $N = 2$

Problem 7

We know that,

$$F\left[e^{-at}\right] = \frac{1}{a + j\Omega}$$

Consider now, $F\left[x(t) = e^{-4t}(u(t) - u(t-2))\right]$,

$$\begin{aligned} F\left[e^{-4t}(u(t) - u(t-2))\right] &= F\left[e^{-4t}u(t) - e^{-4t}u(t-2)\right] \\ &= F\left[e^{-4t}u(t)\right] - F\left[e^{-4t}u(t) - e^{-4(t-2)}u(t-2)e^{-8}\right] \\ &= \frac{1}{4+j\Omega} - \frac{e^{-2j\Omega}}{4+j\Omega}e^{-8}, \text{ using time shift} \\ X(\Omega) &= \frac{1 - e^{-8-2j\Omega}}{4+j\Omega} \end{aligned}$$

Now consider the given signals,

1. $x(t)e^{j5t}$

$$\begin{aligned} F\left[x(t)e^{j5t}\right] &= X(\Omega - 5) \quad \text{(Frequency Shift)} \\ &= \frac{1 - e^{-8-2j(\Omega-5)}}{4 + j(\Omega - 5)} \end{aligned}$$

2. $x(t) + x(t+2)$

$$\begin{aligned} F\left[x(t) + x(t+2)\right] &= X(\Omega) + X(\Omega)e^{j2\Omega} \\ &= (1 + e^{j2\Omega})X(\Omega) \\ &= 2e^{j\Omega}cos\Omega X(\Omega) \end{aligned}$$

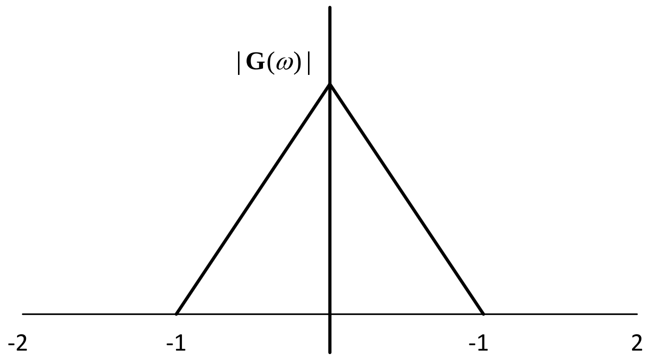
3. $x(t)cos(2t)$

$$\begin{aligned} F\left[x(t)cos(2t)\right] &= F\left[x(t).\frac{(e^{j2t} + e^{-j2t})}{2}\right] \\ &= \frac{1}{2}\left(X(\omega - 2) + X(\Omega + 2)\right) \\ &= \frac{1}{2}\left[\frac{1 - e^{-8-2j(\Omega-2)}}{4 + j(\Omega-2)} + \frac{1 - e^{-8-2j(\Omega+2)}}{4 + j(\Omega+2)}\right] \end{aligned}$$

4. $tx(t)$

$$\begin{aligned} F\left[tx(t)\right] &= j\frac{dX(\Omega)}{d\Omega} \\ &= \frac{1 - 2\Omega j e^{-8-2j\Omega}}{(4 + j2\Omega)^2} \end{aligned}$$

Problem 8



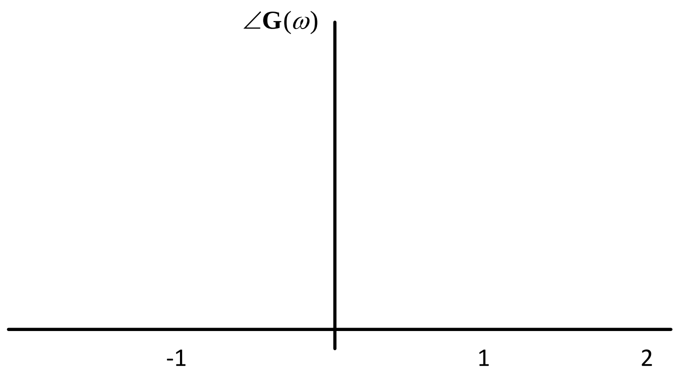


Figure 4: Problem 8. Magnitude and Phase of $G(\omega)$

1. The magnitude and phase is shown in Fig 4.
2. Assuming that $X(\omega)$ is only defined between $\omega \leq 1$

$$\begin{aligned} X(\omega) &= G(\omega) \\ x(t) &= \frac{1}{2} sinc^2\left(\frac{t}{2}\right) \end{aligned}$$

   However, since we have a rectangular function multiplied with the triangle function, the function value outside of the region [-1, 1] can be arbitrary. But they will still be suppressed to zero. If this argument is shown, credit will still be given.

3. Determine $\int\limits_{-\infty}^{\infty} |g(t)|^2 dt$.

$$\begin{aligned} \int\limits_{-\infty}^{\infty} |g(t)|^2 dt &= \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} |G(\omega)|^2 d\omega \\ &= \frac{1}{2\pi}\left(\int_{-1}^{0} (1+\omega)^2 d\omega + \int_{0}^{1} (1-\omega)^2 d\omega\right) \\ &= \frac{1}{3\pi} \end{aligned}$$

Problem 9
The difference equation for the system is

$$y[n] = \frac{1}{3}y[n-1] - \frac{2}{3}y[n-2]$$

Problem 10

1. $x[n] = u[n+3] - u[n-5]$.

$$\begin{aligned} X_d(\omega) &= \sum_{n=-3}^{5} e^{-j\omega n} \\ &= \frac{e^{j\omega 3}\left(1 - e^{j8\omega}\right)}{1 - e^{-j\omega}} \\ &= \frac{e^{j\omega 3} - e^{-j\omega 5}}{1 - e^{-j\omega}} \\ &= \frac{e^{-j\omega}\left(e^{j\omega 4} - e^{-j\omega 4}\right)}{e^{-j\omega/2}(e^{j\omega/2} - e^{-j\omega/2})} \\ &= e^{-j\omega/2}\frac{sin\,(4\omega)}{sin\,(\omega/2)} \end{aligned}$$

2. $x[n] = 2\delta[n+3] + 2\delta[n-3]$

$$\begin{aligned} X_d(\omega) &= 2e^{j\omega 3} + 2e^{-j\omega 3} \\ &= 4cos(3\omega) \end{aligned}$$

3. $x[n] = \left(\frac{1}{3}\right)^{n-4} u[n-4]$

$$X_d(\omega) = \frac{1}{1 - \frac{1}{3}e^{-j\omega}} . e^{-j\omega 4}$$

4. $x[n] = (1-n)2^n u[n]$ This does not converge. Hence Fourier Transform is undefined.

5. The magnitude and phase for Part (1) are shown in Fig 5 and 6 respectively. The magnitude and phase for Part (2) are shown in Fig 7 and 8 respectively.

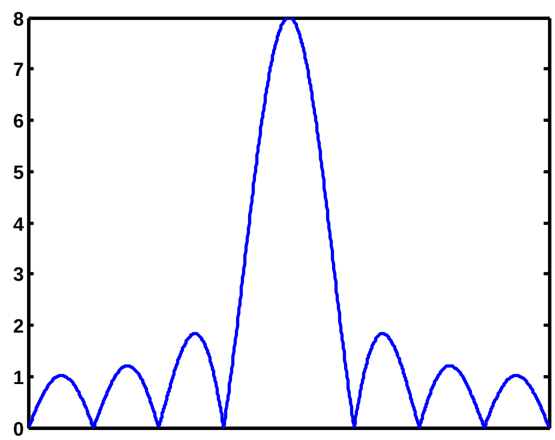


Figure 5: Problem 10. Magnitude for Part (1). Note: the zero crossing points are $(-\pi, -3\pi/2, -\pi/2, -\pi/4, \pi/4, \pi/2, 3\pi/2, \pi)$

Problem 11

1. $X_d(\omega) = 2 + 3e^{-j\omega} + 2e^{-3j\omega} - e^{-5j\omega}$
   Using linearity and time-shift properties of the DTFT:

$$X_d(\omega) = 2 + 3e^{-j\omega} + 2e^{-3j\omega} - e^{-3j\omega} \leftrightarrow 2\delta[n] + 3\delta[n-1] + 2\delta[n-3] - \delta[n-5]$$

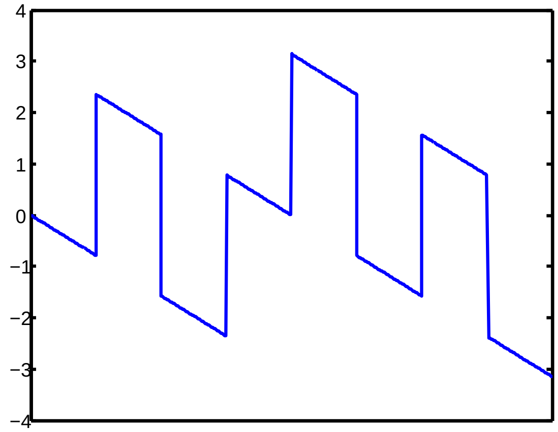


Figure 6: Problem 10. Phase for Part (1). Note: the phase changes at $(-\pi, -3\pi/2, -\pi/2, -\pi/4, \pi/4, \pi/2, 3\pi/2, \pi)$

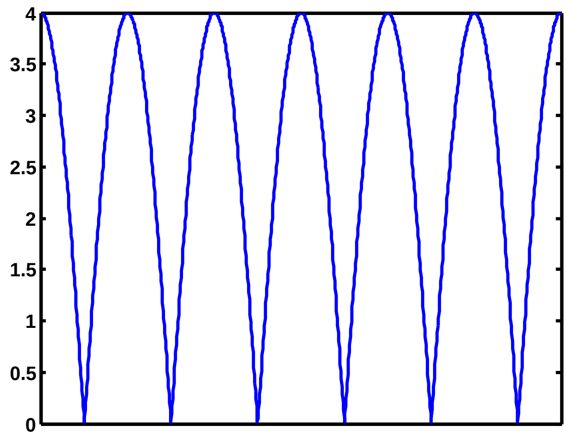


Figure 7: Problem 10. Magnitude for Part (2). Note: the zero crossing points are $(-5\pi/6, -\pi/2, -\pi/6, \pi/6, \pi/2, 5\pi/6)$

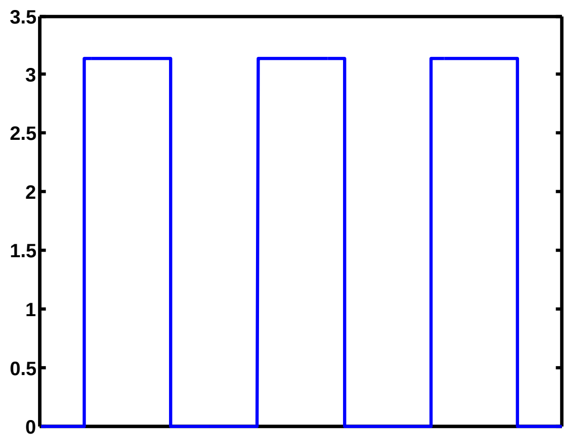


Figure 8: Problem 10. Phase for Part (2). Note:The phase is shown in the interval $(0, \pi)$. The zero crossing points are $(\pi/6, \pi/2, 5\pi/6)$. The phase is a odd function.

2. $X_d(\omega) = \begin{cases} 0, & 0 \leq |\omega| \leq \omega_0 \\ 1, & \omega_0 \leq |\omega| \leq \pi \end{cases}$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{pi} X_d \omega d\omega = \frac{1}{2\pi}\int_{-\pi}^{-\omega_0} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\omega_0}^{\pi} e^{j\omega n} d\omega$$

For $n = 0$,

$$x[0] = \frac{1}{2\pi}(\pi - \omega_0) + \frac{1}{2\pi}(\pi - \omega_0) = \frac{\pi - \omega_0}{\pi}$$

For $n \neq 0$

$$x[n] = \frac{1}{jn2\pi}(e^{-j\omega_0 n} - e^{-j\pi n} + \frac{1}{jn2\pi}(e^{j\pi n} - e^{j\omega_0 n}) = \frac{-sin(n\omega_0)}{n\pi}$$

Thus,

$$x[n] = \delta[n] - \frac{\omega_0}{\pi} sinc(n\omega_0)$$

where,

$$sinc(n) = \begin{cases} \frac{sin(n)}{n}, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

3.

$$\begin{aligned}
x[n] &= \frac{1}{2\pi} \int\limits_{-\pi}^{\pi} X_d \omega e^{j\omega n} d\omega \\
&= \frac{1}{2\pi} \left(\int\limits_{-\pi}^{-9\pi/10} 2e^{j\omega n} d\omega + \int\limits_{-9\pi/10}^{-8\pi/10} e^{j\omega n} d\omega + \int\limits_{8\pi/10}^{9\pi/10} e^{j\omega n} d\omega + \int_{9\pi/10}^{\pi} 2e^{j\omega n} d\omega \right) \\
&= \frac{1}{2\pi} \left(\int_{8\pi/10}^{9\pi/10} (e^{j\omega n} + e^{-j\omega n}) d\omega + \int_{9\pi/10}^{\pi} 2(e^{j\omega n} + e^{-j\omega n}) \right) \\
&= \frac{1}{\pi} \left(\int\limits_{8\pi/10}^{9\pi 10} cos(\omega n) d\omega + \int\limits_{9\pi/10}^{\pi} 2cos(\omega n) d\omega \right), \quad n \neq 0 \\
&= \frac{1}{\pi n} \left[\left(sin\left(\frac{9\pi}{10} n\right) - sin\left(\frac{8\pi n}{10}\right) \right) - \left(sin\left(\pi n\right) - sin\left(\frac{9\pi n}{10}\right) \right) \right], \quad n \neq 0 \\
&= \frac{-1}{\pi n} \left[sin\left(\frac{9\pi}{10} n\right) + sin\left(\frac{8\pi}{10} n\right) \right]
\end{aligned}$$

for $n = 0$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_d(\omega) d\omega = \frac{1}{2\pi} \left(\frac{6\pi}{10}\right) = \frac{3}{10}$$

Hence,

$$x[n] = 2\delta[n] - \frac{9}{10} sinc\left(\frac{9\pi}{10} n\right) - \frac{8}{10} sinc\left(\frac{8\pi}{10} n\right)$$

4. One way is to follow the procedure in part of (3) and solve the inverse DTFT integral. But note that convolving two rectangular functions results in a triangle. Hence the given function $X_d(\omega)$ can be seen as the convolution of $Y(\omega)$ with itself, where in the interval $[-\pi, \pi]$, $X_1(\omega)$ is given by

$$Y(\omega) = \begin{cases} 1, & -\pi/2 \leq \omega \leq \pi/2 \\ 0, & \text{otherwise} \end{cases}$$

We need to solve for $x[n] = F^{-1}[X_d(\omega)]$. We know the following,

$$F(y_1[n]y_2[n]) = \frac{1}{2\pi}[Y_1(\omega) * Y_2(\omega)]$$

If $y_1[n] = y_2[n] = y[n]$,

$$F(y^2[n]) = \frac{1}{2\pi}[Y(\omega) * Y(\omega)]$$

Multiply by 2 and take inverse Fourier transform,

$$2y^2[n] = F^{-1}\left[\frac{1}{\pi}[Y(\omega) * Y(\omega)]\right]$$

or the required time domain signal, x[n] is

$$x[n] = 2y^2[n]$$

where,

$$y[n] = 0.5sinc(0.5n)$$

and,

$$x[n] = 0.5sinc^2(0.5n)$$

Problem 12

1. Evaluate $X_d(\omega)|_{\omega=0}$

$$\begin{aligned} X_d\omega|_{\omega=0} &= \sum_{n=-\infty}^{n=\infty} x[n] \\ &= \sum_{n=-\infty}^{\infty} Re\{x[n]\} + j\sum_{n=-\infty}^{\infty} Im\{x[n]\} \\ &= 12 \end{aligned}$$

2. Evaluate $X_d(\omega)|_{\omega=\pi}$

$$\begin{aligned} X_d\omega|_{\omega=\pi} &= \sum_{n=-\infty}^{\infty} x[n](-1)^n \\ &= (1-3j)-(2+j)+(3-2j)-(3+2j)+(2-j)-(1-3j) \\ &= -12j \end{aligned}$$

3. Evaluate $\int_{-\pi}^{\pi} X_d(\omega)d\omega$

$$\int_{-\pi}^{\pi} = 2\pi\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} X_d(\omega)e^{j\omega 0}d\omega\right) = 2\pi x[0] = 4\pi - j2\pi$$

4. Determine and sketch the signal whose DTFT is $X_d^*(-\omega)$
$X_d^*(-\omega)$ is the DTFT of $x^*[n]$. The signal $x^*[n]$ is shown Fig.9,

Problem 13

1. $x[n] = \delta[n-3], 0 \le n \le 3$

$$\begin{aligned} X[k] &= \sum_{n=0}^{3} \delta[n-3]e^{-j\frac{2\pi}{4}}kn \\ &= e^{-j\frac{3}{2}\pi k} \\ X[k] &= \{1, j, -1, -j\} \end{aligned}$$

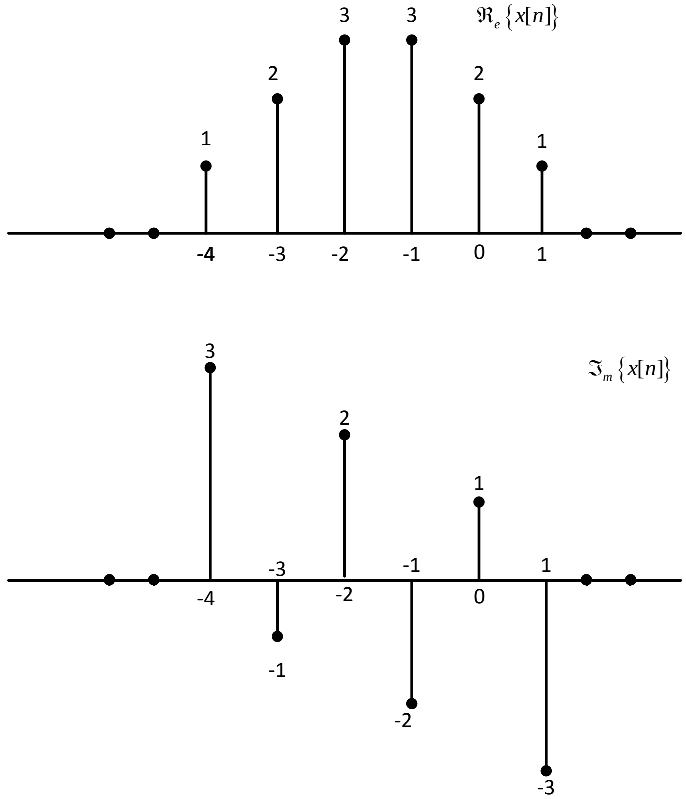


Figure 9: Problem 12. Magnitude and Phase of $X(k)$ in Part (4)

2. $x[n] = \begin{cases} 1 & 0 \leq n \leq 2 \\ 0 & 3 \leq n \leq 5 \end{cases}$

$$X[k] = \sum_{n=0}^{2} e^{-j\pi kn/3}$$

$$X[k] = \begin{cases} 3, & k = 0 \\ \frac{1-e^{-j\pi k}}{1-e^{-j\pi k/3}}, & else \end{cases}$$

$$X[k] = \begin{cases} 3, & k = 0 \\ \frac{e^{-j\pi k/2(sin(\pi k/2))}}{e^{-j\pi k/6} sin(\pi k/6)}, & else \end{cases}$$

3. $x[n] = cos\left(\frac{\pi n}{4}\right), 0 \leq n \leq 7$

$$cos(\frac{n\pi}{4}) = \frac{e^{jn\pi/4} + e^{-jn\pi/4}}{2}$$

$$X[k] = \frac{1}{2}\sum_{n=0}^{7}(e^{jn\pi/4} + e^{-jn\pi/4})e^{-j2\pi n/8}$$

for $k \neq 1, 7$

$$X[k] = 0$$

for $k = 1, 7$

$$X[k] = \frac{1}{2}\left[\frac{1-e^{-j2\pi(k-1)}}{1-e^{-j\pi(k-1)/4}} + \frac{1-e^{-j2\pi(k-1)}}{1-e^{-j\pi(k-1)/4}}\right]$$

Hence,

$$
\begin{aligned}
X[1] &= \frac{1}{2}[8+0] = 4 \\
X[7] &= \frac{1}{2}[0+8] = 4
\end{aligned}
$$

$$
X[k] = 4(\delta(k-1) + \delta(k-7))
$$

4. $x[n] = \begin{cases} 1 & n \text{ even}, \quad 0 \le n \le 6 \\ 0 & n \text{ odd}, \quad 0 \le n \le 6 \end{cases}$

$$
\begin{aligned}
X[k] &= \sum_{m=0}^{\frac{N-1}{2}} e^{-j\frac{2\pi}{N}k2m} \\
&= \sum_{m=0}^{\frac{N-1}{2}} (e^{-j\frac{4\pi k}{7}})^m
\end{aligned}
$$

$$
X[k] = \begin{cases} \frac{N+1}{2}, & k = 0 \\ \frac{1-e^{-j\frac{4\pi}{7}k\frac{N+1}{2}}}{1-e^{-j\frac{4\pi}{7}k}} = \frac{1-e^{-j\frac{16\pi k}{7}}}{1-e^{-j\frac{4\pi}{7}}}, & else \end{cases}
$$

5. Sketch the magnitude and phase for parts (1) and (2).

The magnitude and phase of (1) and (2) are shown in Figs. 10 and 11 respectively.

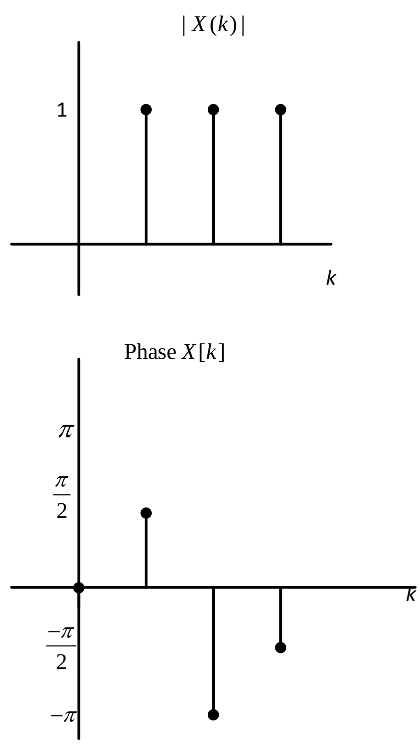


Figure 10: Problem 13. Magnitude and Phase of $X(k)$ in Part (1)

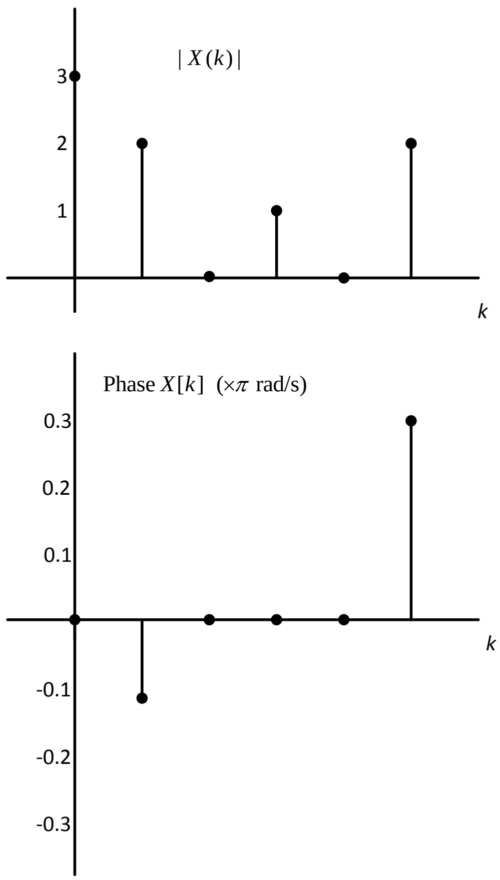


Figure 11: Problem 13. Magnitude and Phase of $X(k)$ in Part (2)