University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 310: Digital Signal Processing I
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Problem Set 2
Summer 2011
Reading: Chapter 2 : Sections 2.5, 2.6 and Chapter 3

Problem 1
Determine if the following systems are **linear** or **non-linear**, **causal** or **non-causal**, **shift-invariant** or **shift-varying**. Assume that the input is zero before $n = 0$ and that the initial conditions of the systems are all set to zero. [*bonus: determine which systems have memory and which are memoryless*]

i. $y[n-2] = x^2[n] + 3y[n]$

ii. $y[n] = x[-n+2]$

iii. $y[n] = \left(\frac{1}{4}\right)^{|n|} x[n]$

iv. $y[n] = \sum_{m=-\infty}^{n+1} x[m]$

v. $y[n] = \frac{x[n]}{x[2]}$

vi. $y[n-1] = x[n-1] + tan(4)x[n] - cos(0.4\pi n)y[n]$

vii. $y[n] = x[12n]$

Problem 2 Evaluate the following expressions

a) Find the inverse DFT of the sequence $X[k] = \{1, e^{-j\pi/2}, 0, e^{j\pi/2}\}$, where the first entry of $X[k]$ corresponds to $k = 0$.

b) Without explicitly computing the inverse DFT sum, find the inverse DFT of the sequence $Y[k] = \{1, -1, 0, -1\}$, where the first entry of $Y[k]$ corresponds to $k = 0$.

Problem 3 Given the eight-point DFT of the sequence

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 3 \\ 0, & 4 \leq n \leq 7 \end{cases}$$

compute the DFT of the following sequences (i.e., give your answer in terms of $X[k]$).

a) $x_1[n] = \begin{cases} 1, & n = 0 \\ 0, & 1 \leq n \leq 4 \\ 1, & 5 \leq n \leq 7 \end{cases}$

b) $x_2[n] = \begin{cases} 0, & 0 \le n \le 2 \\ 1, & 3 \le n \le 6 \\ 0, & n = 7 \end{cases}$

Problem 4 Let $x[n] = (0.9)^n u[n]$, find the discrete-time convolution of the following:

a. $y_1[n] = x[n] * h[n]$, where $h[n] = (0.7)^n u[n]$

b. $y_2[n] = x[n] * w[n]$, where $w[n] = u[n] - u[n-10]$

c. $y[n] = x[n] * v[n]$, where $v[n] = \frac{1}{5}h[n-1] + \frac{1}{10}w[n+9]$

d. $y_0[n] = s[n] * t[n]$, where $s[n] = x[n-3]$ and $t[n] = h[n+1]$

[*hint: for parts (c) and (d), there is an easier way to compute the discrete-time convolution than using the convolution sum directly*]

Problem 5 Given a causal, linear shift-invariant system characterized by,

$$y[n] - \frac{1}{4}y[n-1] = 4x[n] + 3x[n-1],$$

find the impulse response $h[n]$ (i.e., determine $y[n]$ when $x[n] = \delta[n]$).

Problem 6 Find a closed-form expression for the output $y[n]$ of the system described by the block diagram in Fig. 1, for the input $x[n] = 0.5^n u[n-2]$. Assume the system is at rest at time $n = 0$. [*hint: you have already obtained the impulse response of this system*]

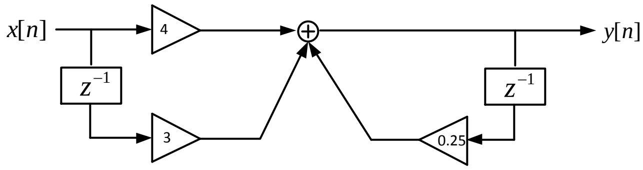

Figure 1: Problem 6.Block diagram for Problem 6

Problem 7 The continuous time signal $x_a(t)$ has the Fourier transform shown in Fig. 2. The signal $x_a(t)$ is sampled with a sampling period of T to produce the discrete-time signal $x[n] = x_a(nT)$.

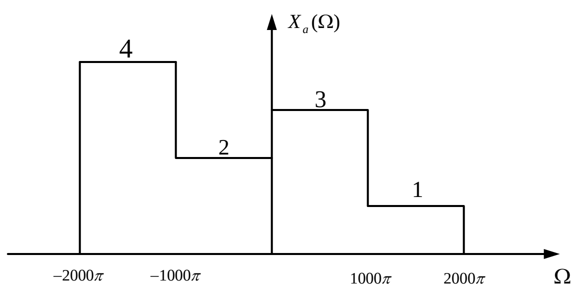

Figure 2: Problem 7. Fourier Transform $X(\Omega)$ of $x_a(t)$

a. Sketch the DTFT ($X_d(\omega)$) of the signal above for the sampling frequencies F_s of (i) 2000 Hz, (ii) 3000 Hz.

b. What is the Nyquist rate for the signal $x_a(t)$?

Problem 8 Let $x_a(t) = cos(4\pi t) + 0.15cos((63/16)\pi t)$. Let N denote the number of samples acquired and T be the sampling rate when $x_a(t)$ is sampled such that $x[n] = x_a(nT), n = 0, 1, \ldots, N-1$.

a. What is the Nyquist rate for the signal $x_a(t)$?

b. If $N = 128, T = \frac{1}{10}$s, and a 512-point DFT $\{X[k]\}_{k=0}^{511}$ is computed, which sample(s) would have the greatest amplitude?

c. If $F_s = 10$ Hz, (where F_s is the sampling frequency), what is the minimum value of N required in order to resolve the two frequency components in $x_a(t)$ (state and justify the criteria used for your answer)?

d. Using the value of N that you calculated in part (c), what value of T (and what sampling frequency F_s) would give your spectral analysis method the highest resolution? What would be this method's resolution (i.e., the minimum difference in analog frequencies that can be resolved)?

e. Intuitively (DO NOT GIVE AN EQUATION), why does increasing the sampling frequency (F_s) degrades the resolution (i.e. the minimum resolvable frequency is increased)?

Problem 9 The continuous-time signal:

$$x_a(t) = sin(15\pi t) + cos(90\pi t)$$

is sampled with a period T to obtain the following discrete-time signal:

$$x[n] = sin(\frac{\pi}{5}n) + cos(\frac{6\pi}{5}n)$$

a. Determine a choice for T consistent with the continuous-time signal and its sampled discrete-time counterpart.

b. Is this T found in part a unique? If so, expound. If not, specify another choice of T consistent with the signals.

Problem 10 The continuous-time signal:

$$x_a(t) = cos(450\pi t)$$

is sampled with a period T to obtain the following discrete-time signal:

$$x[n] = x_a(nT)$$

a. Compute and sketch the magnitude of the continuous-time Fourier transform of $x_a(t)$ and the discrete-time Fourier transform of $x[n]$ for $T = 1$ms and 2.5ms.

b. Find the maximum sampling period T_{max} such that aliasing does not occur.

Problem 11 Solve the difference equation you developed in Homework 1, Problem 9.

***Reminder - Homework is due on 07/01/2011 at 5:00 PM - place your assignments in the <u>ECE 410</u> homework drop box in Everitt Hall!**