University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 310: Digital Signal Processing I
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Problem Set 3
Summer 2011
Reading: Chapter 5

Problem 1
Find the 1-sided z-transforms and regions of convergence (ROC)s for each of the following sequences. Recall that $u[n]$ is the unit step sequence, i.e.,

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

a) $x[n] = \left(\frac{1}{3}\right)^n u[n]$

b) $x[n] = \left(\frac{1}{3}\right)^n u[n-3]$

c) $x[n] = n^2 u[n]$

d) $x[n] = e^{\frac{j\pi n}{3}} u[n]$

e) $x[n] = sin(\omega n + \theta) u[n]$

f) $x[n] = n\left(\frac{1}{2}\right)^n u[n]$

Problem 2
Find the inverse unilateral z-transform of each the following 1-sided z-transforms with associated ROCs:

a) $\frac{z^2 - z}{z^2 + 3z + 2}, |z| > 2$

b) $\frac{z^2}{z^2 + z + 0.5}, |z| > \frac{\sqrt{2}}{2}$

Problem 3
Find the two-sided Z-transform, including the region of convergence, for each of the following sequences.

a) $x[n] = \left(\frac{1}{2}\right)^n$

b) $x[n] = \left(\frac{1}{2}\right)^{|n|}$

c) $x[n] = \delta[n]$

d) $x[n] = \left(\frac{1}{3}\right)^n u[-n+1]$

e) $x[n] = \begin{cases} n, & 0 \leq n \leq N-1 \\ N, & n \geq N \end{cases}$

Problem 4
Determine all possible regions of convergence of $Y(z)$, where

a) $Y(z) = X_1(z) + X_2(z), x_1[n] = a^n u[n],$ and $x_2[n] = b^n u[n-3],$ where $|a| > |b|$

b) $y[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m],$ where $H(z) = \frac{1}{z^2+6z+8},$ and $x[n] = 2\delta[n] - \delta[n-3]$

Problem 5
Use the unilateral z-transform to find the output $y[n]$ of the system in Fig. 1, when the input $x[n]$ is given by $x[n] = b^n$, $n \geq 0$, with $y[-1] = 1$, and $|a| < |b| < 1$.

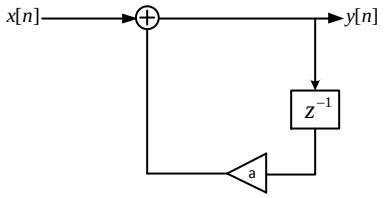

Figure 1: System for Problem 5

Problem 6
Let $h[n]$ be the unit pulse response of a discrete-time LSI system,

$$h[n] = \begin{cases} 1, & -1 \leq n \leq 10 \\ \left(\frac{1}{3}\right)^n, & n \geq 11 \end{cases}$$

Find the output of the system when the input is $x[n]$ given by,

$$x[n] = \begin{cases} 2^n, & n \leq 2 \\ \left(\frac{1}{2}\right)^n, & n \geq 3 \end{cases}$$

Find the output two ways:

a) First find the output in the time-domain, using the convolution sum.

b) Then find $y[n]$ using the two-sided Z-transform.

Problem 7
Given a linear time invariant system characterized by,

$$\frac{z^2 - \frac{1}{2}}{z^2 - \frac{3z}{4} + \frac{1}{8}}, \text{ ROC } |z| > \frac{1}{2}$$

find the impulse response of the system. Is the system causal? What is the difference equation relating the input $x[n]$ to the output $y[n]$ for this system? Assume that the system is initially at rest before the input is applied.

Problem 8
Sketch the pole-zero plot of

$$X(z) = \frac{z^2 + z + 2}{z^2 + z - 2}$$

Show all possible regions of convergence and find the corresponding sequences $x[n]$ for each possible region of convergence.

Problem 9
Given $h[n] = (1/5)^n u[n]$ and input $x[n] = (1/2)^n u[n-2]$, calculate $y[n]$ by computing

a) The convolution sum.

b) By evaluating $Y(z)$ from $H(z)$ and $X(z)$ and then taking the inverse z-transform.

Confirm that the two methods give you the same $y[n]$.

Problem 10

A system is described by,

$$y[n] = 0.5y[n-1] - 0.04y[n-2] + x[n] + x[n-1]$$

Assuming zero initial conditions, evaluate $y[n]$ given $x[n] = (1/4)^n u[n]$ using z-transforms.

Problem 11

Determine the transfer function and impulse response of a system described by,

$$y[n] = 0.5y[n-1] - 0.06y[n-2] + 4x[n] + 8x[n-1]$$

Problem 12

You wish to take out a mortgage for a house valued at P dollars with no down payment. The principle for your mortgage is then P dollars. If the term of the loan is N months, and interest is compounded monthly at an annual percentage rate of R, then after each monthly payment of X dollars, you will owe $y[n]$ dollars, i.e., the balance of the loan satisfies the following difference equation,

$$y[n] = (1+r)(y[n-1] - x[n])$$

where $r = R/12$ and $x[n] = X$ is your monthly payment during the period of the loan. Use 1-sided z-transforms to find the monthly payment for a \$300,000 loan over 30 years with a fixed APR of 8%. Hint: What are the initial conditions for $y[n]$, i.e., what is $y[0]$? What are the final conditions, i.e. what is $y[30 * 12]$?

***Reminder - Homework is due on 07/08/2011 at 5:00 PM - place your assignments in the ECE 410 homework drop box in Everitt Hall!**