University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 310: Digital Signal Processing I
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Problem Set 4
Summer 2011
Reading: Chapter 5: Sections 5.10-5.14, Chapter 8: Frequency response of LSI systems

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Problem 1
Determine whether each of the following represents a BIBO stable system:

1. $H(z) = \frac{z-7}{z^2+1/9}$, causal
2. $H(z) = \frac{z-7}{z^2+1/9}$, anticausal
3. $H(z) = \frac{z}{(z-0.7)(z^2+z+1)}$, $h[n]$ is two-sided
4. $H(z) = \frac{z+1}{z-1}$, causal
5. $H(z) = \frac{z-1}{z^2+j}$, causal

For each case in which the system is determined to be unstable, find a bounded real-valued input that will produce an unbounded output.

Problem 2
For the block diagram shown in Fig. 1:

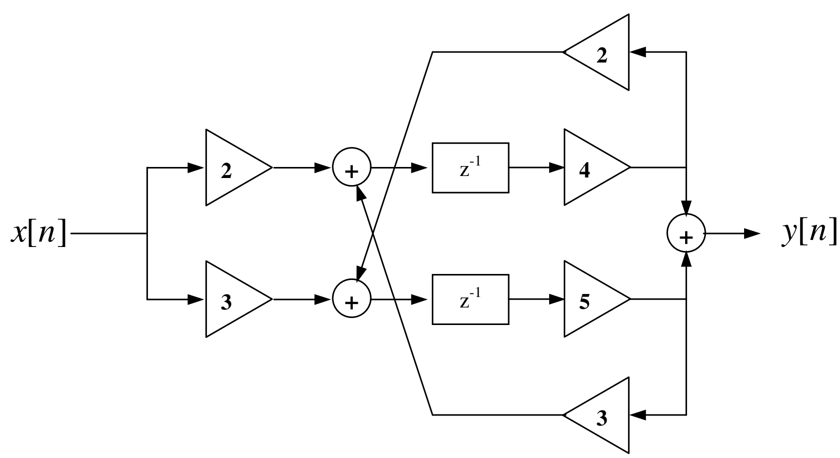

Figure 1: System for Problem 2

a. write down the difference equation.

b. draw the block-diagram of a direct-form II architecture that implements the same function as the block diagram shown above.

For each case in which the system is determined to be unstable, find a real-valued input that will produce an unbounded output.

Problem 3

For the difference equation $0 = 2y[n+2] - 2y[n] + 4x[n+2] + 2x[n+1]$,

a. Put the difference equation in standard causal form $y[n] = \cdots$

b. Draw a block diagram (i.e., flow diagram, or digital filter structure) implementing this difference equation.

c. For the input $x[n] = \delta[n]$ and the system at rest (i.e., zero initial conditions), compute $y[-1]$, $y[0]$, $y[1]$, $y[2]$, and $y[3]$ by hand directly from the difference equation.

d. Find the transfer function $H(z)$.

e. Compute the unit pulse response, $h[n]$, via inverse z-transform. Confirm that $h[-1]$ through $h[3]$ are identical to the samples of the unit pulse response computed above via direct iteration of the difference equation.

Problem 4

An LSI system is described by the difference equation

$$y[n] = x[n] + x[n-10]$$

1. Compute and sketch its magnitude and phase response
2. Determine its output to inputs

   (a) $x[n] = \cos\frac{\pi}{10}n + 3\sin\left(\frac{\pi}{3}n + \frac{\pi}{10}\right)$

   (b) $x[n] = 10 + 5\cos\left(\frac{2\pi}{5}n + \frac{\pi}{2}\right)$

Problem 5

Two systems with unit-pulse responses

$$h_1[n] = \frac{4}{7}(2)^{n-1}u[-n] + \frac{11}{7}\left(\frac{1}{4}\right)^{n-1}u[n-1], \qquad h_2[n] = \delta[n] - 3^{n-1}u[-n]$$

are connected in cascade. For each of the individual systems, as well as for the cascade, determine whether it is causal and/or BIBO stable.

Problem 6

The response of a real LSI system for input

$$x[n] = 3 + \cos\left(\frac{\pi}{4}n + 10^\circ\right) + \sin\left(\frac{\pi}{3}n + 25^\circ\right)$$

is

$$y[n] = 9 + 2\sin\left(\frac{\pi}{4}n + 10^\circ\right) .$$

Determine the system response $\tilde{y}[n]$ for input

$$\tilde{x}[n] = 5 + 2\sin\left(\frac{\pi}{4}n + 15^\circ\right) + 10\cos\left(-\frac{\pi}{3}n + 25^\circ\right) .$$

Problem 7

The frequency response of an LSI system is

$$H_d(\omega) = \omega e^{j\sin(\omega)}, \qquad |\omega| \leq \pi .$$

Determine the system output $y[n]$ for the following inputs:

a. $x[n] = 5 + 10e^{j(\frac{\pi}{4}n+45^\circ)} + j^n$

b. $x[n] = 5 + 10\cos(\frac{\pi}{4}n + 45^\circ) + j^n$.

Problem 8

The input $x[n] = 2^n\,(u[n] - 3u[n-1])$ to an unknown LSI system produces output $y[n] = (3^n - 2^n)u[n]$.

a. Determine the unit pulse response $h[n]$.

b. Is the solution unique? What if the system is known to be unstable? What if it is causal?

Problem 9

Consider a filter with the system function

$$H(z) = b_0 \frac{(1 - e^{j\omega_0}z^{-1})(1 - e^{-j\omega_0}z^{-1})}{(1 - re^{j\omega_0}z^{-1})(1 - re^{-j\omega_0}z^{-1})}$$

a. Sketch the pole-zero pattern.

b. From the pole-zero pattern, using geometric arguments, show that for $r \approx 1$, the system is a notch filter and provide a rough sketch of its magnitude response if $\omega_0 = 60^\circ$.

c. For $\omega_0 = 60^\circ$ choose b_0 so that the maximum value of $|H(\omega)|$ is 1.

d. Draw a direct form II realization of the system.

Problem 10

There are three causal systems with the following difference equations labeled (1), (2), and (3):

$$\frac{3}{2}y[n] - \frac{7}{2}y[n-1] + y[n-2] = 2x[n] + x[n-1] \quad (1)$$

$$4y[n] + 2y[n-1] = x[n] - \frac{7}{12}x[n-1] + \frac{1}{12}x[n-2] \quad (2)$$

$$-y[n] + \frac{3}{10}y[n-1] + \frac{1}{10}y[n-2] = 2x[n] - 4x[n-1] \quad (3)$$

a. Find the expression for the transfer function of the all three systems, $H_1(z)$, $H_2(z)$, $H_3(z)$, and plot their pole-zero graphs, and determine whether each of them is BIBO stable or not.

b. Two of these Filters are then connected in cascade, so that the cascaded system is BIBO stable. Select the two systems that will result in a BIBO stable system when connected in cascade. Find the expression for this system's transfer function, and then plot its pole-zero graph.

c. Find the difference equation of the cascaded system.

Problem 11 The pole-zero plots of four causal systems labeled A, B, C, and D are plotted in Figures on the pages attached. Zeros are marked with an 'o' and poles are marked with an 'x'.

a. Without finding the expressions of the transfer functions of the systems, determine which of the systems are BIBO stable.

b. Find the expressions for all the transfer functions $H_A(z)$, $H_B(z)$; $H_C(z)$; and $H_D(z)$.

c. Cascade the two systems B and C. Find the expression of the transfer function $H_B(z)H_C(z)$, determine whether it is BIBO stable, and find its inverse z-transform, $h_{BC}[n]$.

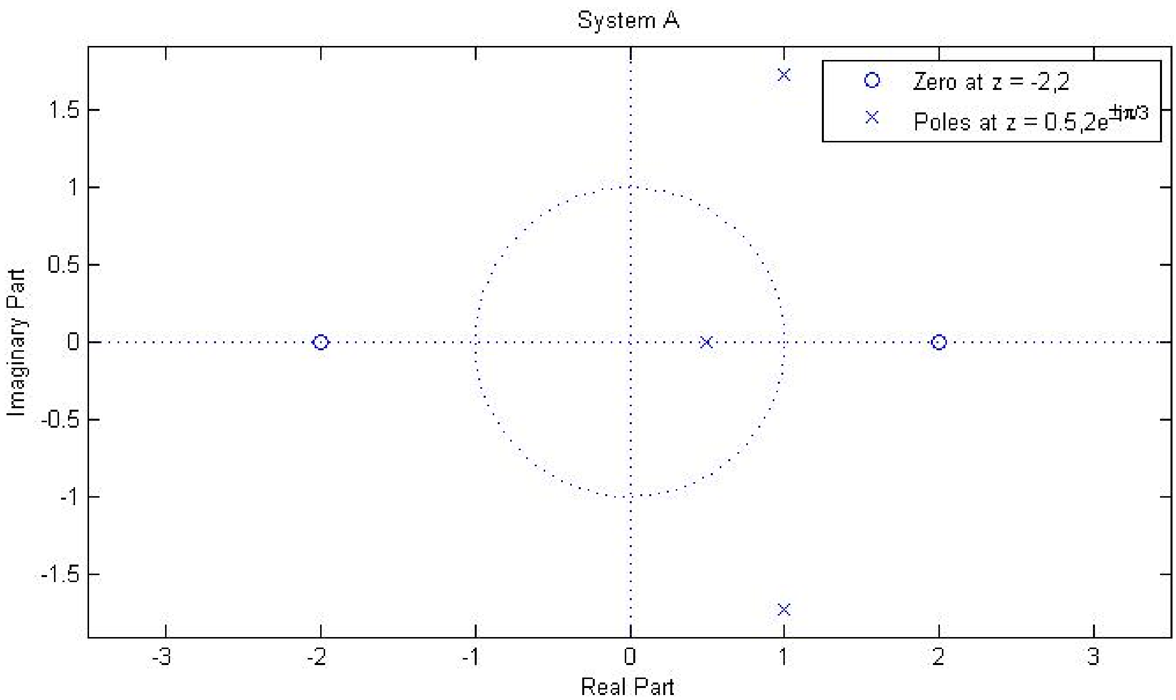

Figure 2: System A for Problem 11

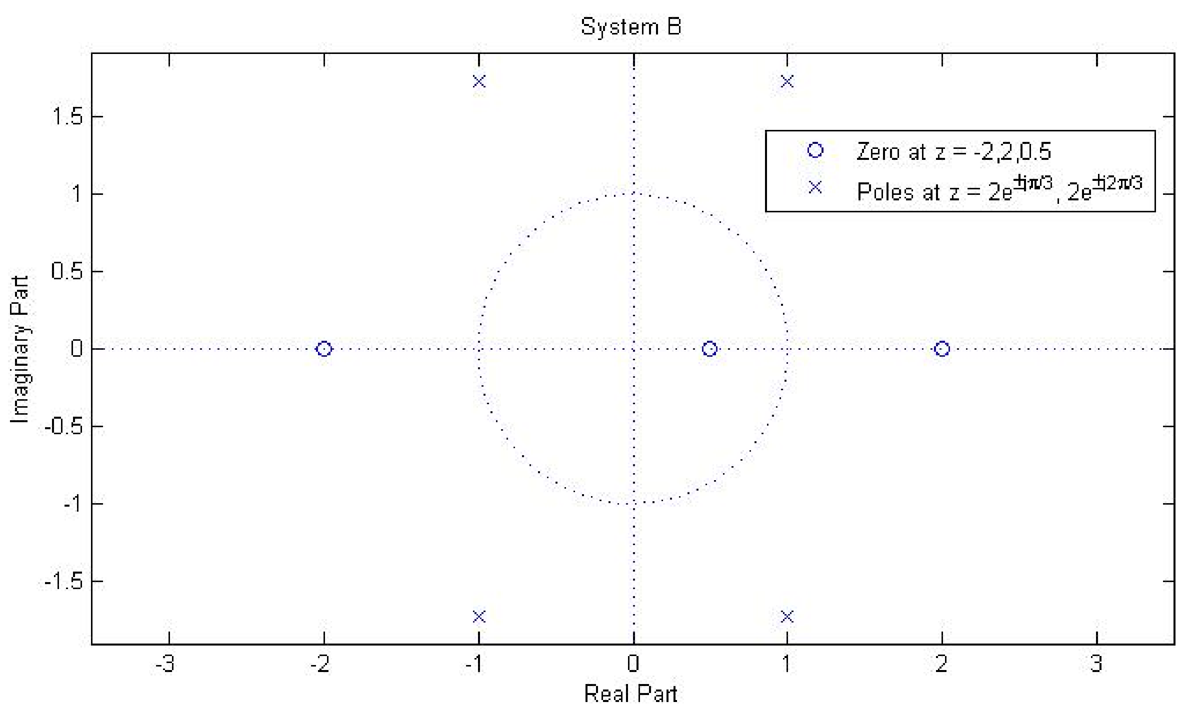

Figure 3: System B for Problem 11

d. Cascade the two systems A and D. Find the expression of the transfer function $H_A(z)H_D(z)$, determine whether it is BIBO stable, and find its inverse z-transform, $h_{AD}[n]$.

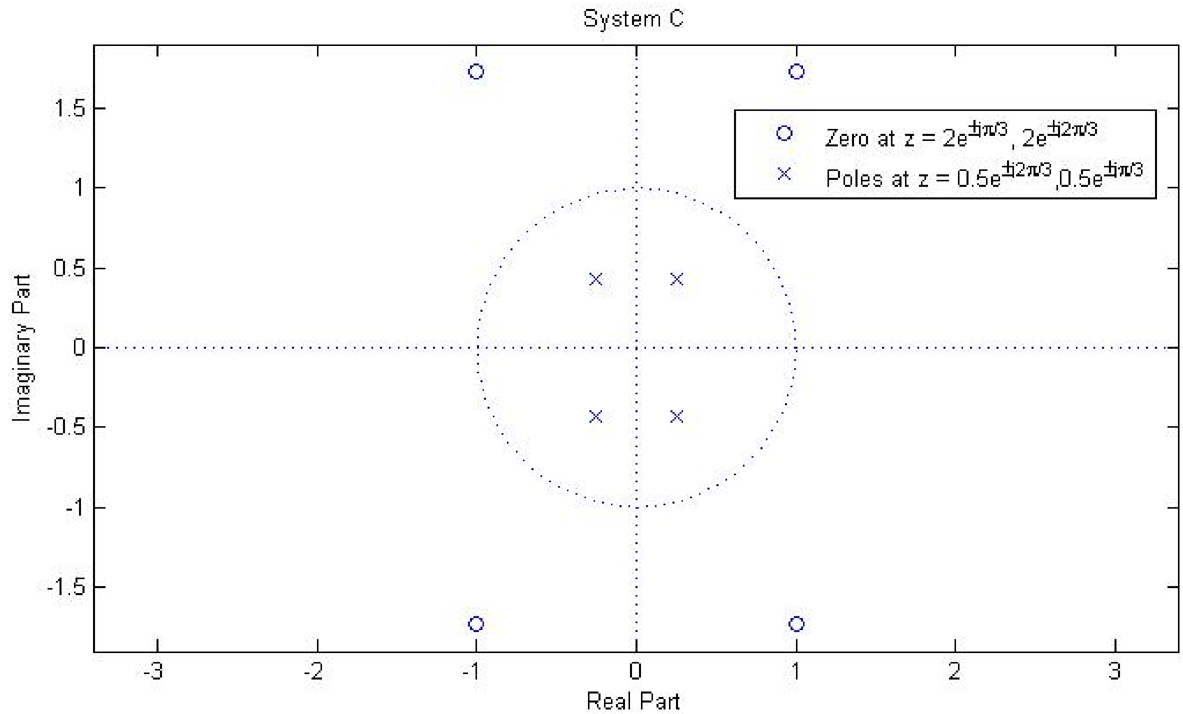

Figure 4: System C for Problem 11

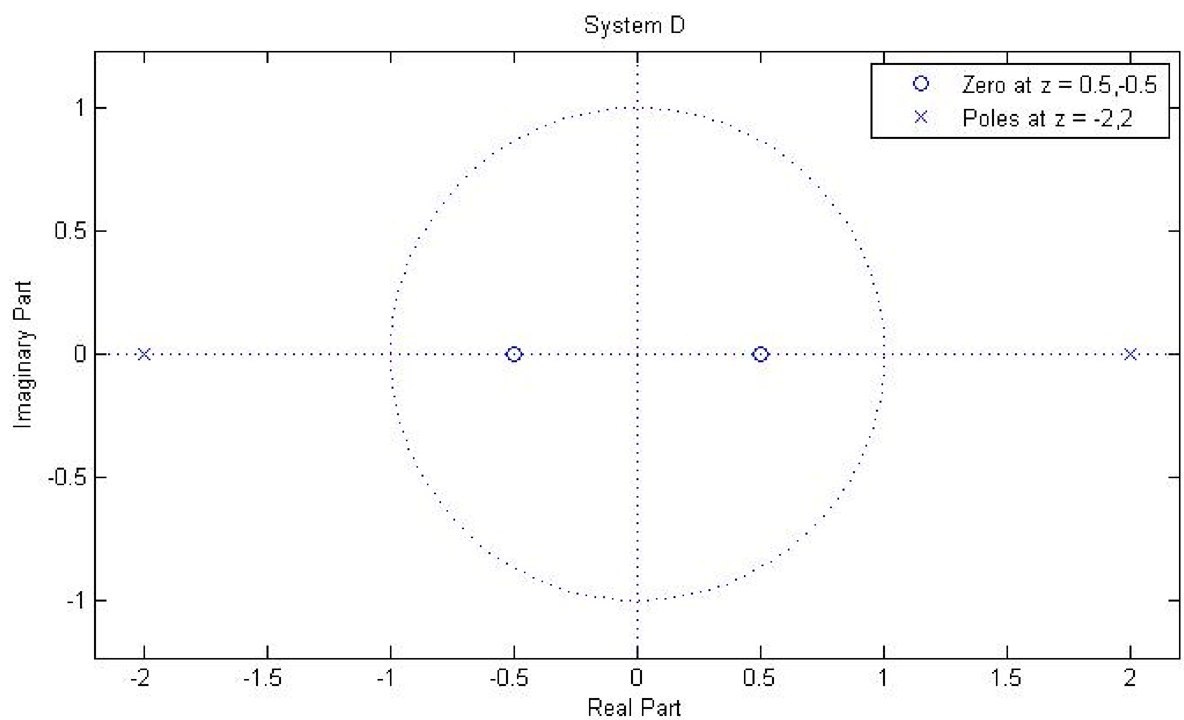

Figure 5: System D for Problem 11

***Reminder - Homework is due on 07/15/2011 at 5:00 PM - place your assignments in the <u>ECE 410</u> homework drop box in Everitt Hall!**