University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 310: Digital Signal Processing I
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Problem Set 5
Summer 2011
Reading: Chapter 8: Frequency response of LSI systems, Chapter 9: A/D and D/A Converters

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Problem 1
Suppose that a bandlimited signal $x_a(t)$ is sampled above the Nyquist rate to give $x[n] = x_a(nT)$, $-\infty < n < \infty$. Based on

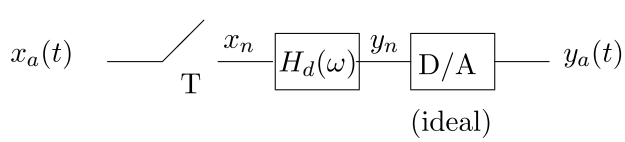
$$X_d(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X_a\left(\frac{\omega + 2\pi n}{T}\right)$$

and assuming an ideal D/A, derive the formula that gives $x_a(t)$ in terms of its samples $x[n]$.

Problem 2
A speech signal $x_a(t)$ is assumed to be bandlimited to 12kHz. It is desired to filter this signal with a bandpass filter that will pass the frequencies between 300Hz and 6kHz by using a digital filter $H_d(\omega)$ sandwiched between an A/D and an ideal D/A.

1. Determine the Nyquist sampling rate for the input signal.
2. Sketch the frequency response $H_{d,1}(\omega)$ for the necessary discrete-time filter, when sampling at the Nyquist rate.
3. Find the largest sampling period T for which the A/D, digital filter response ($H_{d,2}(\omega)$), and D/A can perform the desired filtering function. (Hint: some amount of aliasing may be permissible during A/D conversion for this part.)
4. For the system using T from part (c), sketch the necessary $H_d(\omega)$.

Problem 3
Consider the following system with uniform sampling



The discrete-time system $H_d(\omega)$ is an ideal low-pass filter with cutoff frequency $\frac{\pi}{8}$.

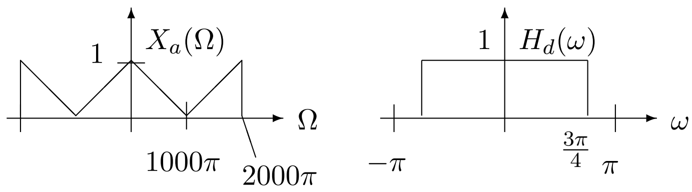
(a) If $x_a(t)$ is bandlimited to 5 kHz, what is the maximum value of T that will avoid aliasing in the A/D converter?

(b) If $\frac{1}{T} = 10$ kHz and $x_a(t)$ is sufficiently bandlimited such that the overall system from $x_a(t)$ to $y_a(t)$ behaves as an LTI system, what will the cutoff frequency of the effective continuous-time filter be?

(c) Repeat part (b) for $\frac{1}{T} = 20$ kHz.

Problem 4

For the digital system in problem 3, assume $T = 0.5$ msec, with



(a) Sketch $X_d(\omega)$, $Y_d(\omega)$, and $Y_a(\Omega)$.

(b) Suppose the ideal D/A is now replaced by a zero-order hold, using the pulse

$$p_a(t) = \begin{cases} 1, & 0 \leq t \leq T \\ 0, & \text{else.} \end{cases}$$

Sketch $Y_a(\Omega)$ for $|\Omega| \leq 8000\pi$. Find the amplitude of the largest unwanted (out of the band $|\Omega| \leq \frac{\pi}{T}$) component of $Y_a(\Omega)$, due to the nonideal D/A.

Problem 5

Let $x(t)$ be a continuous-time signal with samples $x(nT)$ and consider a zero-order hold (ZOH) operation with an effective sampling period T to produce a signal

$$x_0(t) = \sum_{n=-\infty}^{+\infty} x(nT)p_0(t - nT)$$

Let $x_1(t)$ be the result of a first-order hold (FOH) operation on the samples $x(nT)$ of the signal $x(t)$, i.e.,

$$x_1(t) = \sum_{n=-\infty}^{+\infty} x(nT)p_1(t - nT)$$

where

$$p_1(t) = \begin{cases} 1 - \frac{t}{T}, & 0 \leq t \leq T \\ 1 + \frac{t}{T}, & -T \leq t \leq 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Specify the frequency response of a continuous-time filter that produces $x_1(t)$ as its output when $x_0(t)$ is the input.

(b) The FOH can be followed with an appropriate analog filter to form an ideal D/A. Find the frequency response of the analog filter.

(c) How does the frequency response of the filter in part (b) compare with that filter needed to follow a ZOH?

Problem 6

You are given the task of implementing an analog double-echo generator using digital system. The echo generator is specified by the following system equation:

$$y_a(t) = x_a(t) + \alpha\, x_a(t - \tau_d) + \beta\, x_a(t - 2\tau_d)$$

where $x_a(t)$ and $y_a(t)$ are the input and output of the analog system, respectively, and τ_d is the time delay constant. The implementation uses the system shown in Figure 1 (with ideal C/D and D/C) for discrete-time processing of analog signals.

Assume $x_a(t)$ is bandlimited to 20 KHz.

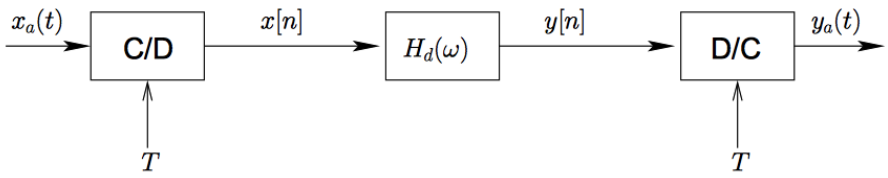


Figure 1: System for Problems 1 and 2. The C/D and D/C convertors are ideal.

(a) Find the desired analog frequency response $H_a(\Omega)$. (Your answer will be in terms of τ_d, α, and β, which are assumed to be known.)

(b) Find the appropriate sampling period $T = T_0$.

(c) Find the digital filter response $H_d(\omega)$ that is needed to implement the analog system.

(d) Assuming that $\tau_d = 100T_0$, find the transfer function $H(z)$ corresponding to $H_d(\omega)$, and draw a block diagram for its implementation.

Problem 7
Consider the following system in Fig. (2) Sketch and label the Fourier Transform of $y_c(t)$ for each of the following cases:

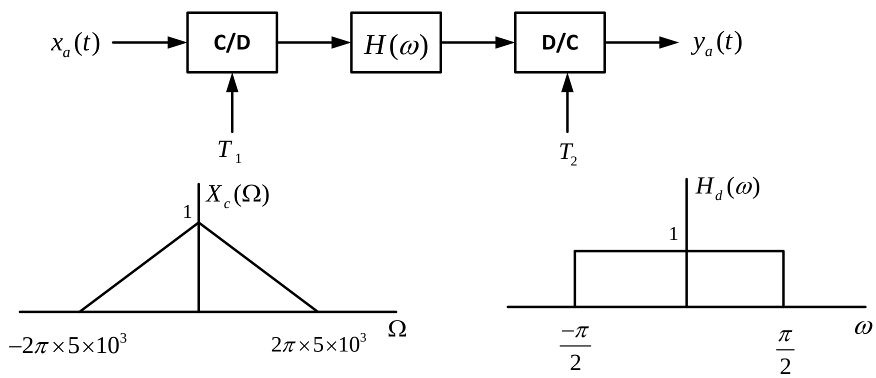


Figure 2: System for Problem 7.

(a) $1/T_1 = 1/T_2 = 10^4$

(b) $1/T_1 = 1/T_2 = 2 \times 10^4$

(c) $1/T_1 = 2 \times 10^4, 1/T_2 = 10^4$

(d) $1/T_1 = 10^4, 1/T_2 = 2 \times 10^4$

Problem 8
The transfer functions of three LSI systems are given below. For each system, determine whether it is an FIR or IIR filter.

(a) $\frac{z^2+3z+2}{2z^2+3z-1}$

(b) $\frac{z+1}{z^2-\frac{z}{4}-\frac{1}{8}}$

(c) $2 + z^{-1} - \frac{1}{3}z^{-2}$

***Reminder - Homework is due on 07/22/2011 at 5:00 PM - place your assignments in the ECE 410 homework drop box in Everitt Hall!**