University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 310: Digital Signal Processing I
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Problem Set 6
Summer 2011
Reading: Chapter 10-13: FIR Filter Design, IIR Filter Design, Digital Interpolation

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Problem 1
Determine the coefficients of a linear-phase FIR filter

$$y[n] = h_0 x[n] + h_1 x[n-1] + h_2 x[n-2]$$

such that (i) it rejects any frequency component at $\omega_0 = 5\pi/6$, and (ii) its frequency response is normalized so that $H_d(0) = 1$.

Problem 2
The frequency response of a GLP filter can be expressed as $H_d(\omega) = R(\omega)e^{j(\alpha - M\omega)}$ where $R(\omega)$ is a real function. For each of the following filters, determine whether it is a GLP filter. If it is, find $R(\omega)$, M, and α, and indicate whether it is also a linear phase filter.

1. $\{h_n\}_{n=0}^{2} = \{2, 1, 1\}$
2. $\{h_n\}_{n=0}^{2} = \{1, 2, 3\}$
3. $\{h_n\}_{n=0}^{2} = \{-1, 3, 1\}$
4. $\{h_n\}_{n=0}^{4} = \{1, 1, 1, -1, -1\}$
5. $\{h_n\}_{n=0}^{2} = \{1, 0, -1\}$
6. $\{h_n\}_{n=0}^{3} = \{2, 1, 1, 2\}$

In each case, the remaining terms of the unit pulse response of the filter are zero.

Problem 3

(a) Determine whether the system is FIR or IIR? If the system is FIR, sketch its $h[n]$.

(b) If the system is FIR, determine whether $h[n]$ is of even or odd length.

(c) If the system is FIR, determine whether $h[n]$ has even or odd symmetry, or neither.

(d) Determine whether its $H_d(\omega)$ has linear phase. If it does not, determine whether it has Type I or Type II generalized linear phase (GLP), or neither.

(e) The frequency response of a GLP filter can be expressed as $H_d(\omega) = R_d(\omega)\, e^{j(\alpha - M\omega)}$, where $R_d(\omega)$ is a real function. For systems that have a GLP or linear phase, find $R_d(\omega)$, M, and α.

(i) $y[n] = \frac{2}{5}x[n] - x[n-1] + x[n-2] - \frac{2}{5}x[n-3]$

(ii) $y[n] = \frac{1}{3}x[n] + x[n-1] - \frac{3}{4}x[n-2]$

(iii) $y[n] = x[n] + x[n-2] + x[n-4]$

Problem 4

Use the windowing method to design a length-N low-pass, generalized linear phase FIR filter with cut-off frequency $\pi/4$.

1. Find an expression for the filter coefficients $\{h_n\}_{n=0}^{N-1}$ if the rectangular window is used for the design.
2. Find an expression for the filter coefficients $\{h_n\}_{n=0}^{N-1}$ if the Hamming window is used for the design.

Problem 5

Use the frequency sampling method to design a length-100 FIR high-pass filter having cutoff frequency $3\pi/4$.

1. Should the filter have type-1 GLP or type-2 GLP?
2. Find an expression for the filter coefficients $\{h_n\}_{n=0}^{99}$.

Problem 6

The bilinear transform is to be used with the analog prototype $H_L(s) = \frac{s}{s+2}$ to determine the transfer function $H(z)$ of a digital HPF with 3 dB cutoff $\pi/3$ (i.e., $|H_d(\pi/3)|^2 = 0.5$).

1. Determine the 3 dB cutoff for the analog prototype Ω_c.
2. Find $H(z)$ in closed form.

Problem 7

For the following bilinear transformation

$$s = \frac{1 - z^{-1}}{1 + z^{-1}} \quad \text{or} \quad z = \frac{1 + s}{1 - s}$$

four points are given in the s-plane as follows: (a) $s_1 = j$, (b) $s_2 = 2$, (c) $s_3 = -2$, and (d) $s_4 = 0$. Find the corresponding z-plane points and mark their locations in the z-plane.

Problem 8

The transformation $s = 2(1 - z^{-1})/(z^{-1} + 1)$ was applied to an analog prototype to design a HPF with a cutoff at $3\pi/5$. The width of the transition band of the resulting digital filter, from stopband edge to cutoff, is $\pi/10$. What is the corresponding transition bandwidth of the analog prototype?

Problem 9 An ideal analog integrator is described by the system function $H_L(s) = 1/s$. A digital integrator with system function $H(z)$ can be obtained using the bilinear transformation, that is,

$$H(z) = \frac{T}{2}\left(\frac{1 + z^{-1}}{1 - z^{-1}}\right) = H_L(s)\big|_{s=(2/T)(1-z^{-1})/(1+z^{-1})}$$

1. Write the difference equation for the digital integrator relating the input $x[n]$ to the output $y[n]$.
2. Roughly sketch the magnitude $|H_a(\Omega)|$ and phase $\angle H_a(\Omega)$ of the analog integrator.

3. Show that the frequency response of the digital integrator is

$$H_d(\omega) = -j\frac{T}{2}\left(\frac{\cos(\omega/2)}{\sin(\omega/2)}\right) = -j\frac{T}{2}\ \text{ctan}\left(\frac{\omega}{2}\right)$$

4. The digital integrator has a pole at $z = 1$. If you implement this filter on a digital computer, what restrictions might you place on the input signal sequence $x(n)$ to avoid computational difficulties?

5. $H_L(s)$ only has a pole. This pole was mapped by the BLT to a pole in $H(z)$. Where did the zero in $H(z)$ come from?

Problem 10

A compact disc player reproduces an audio signal having a 20 kHz bandwidth from samples collected at the rate of 44,100 samples per second. If the D/A converter uses oversampling by a factor of L, then for each value of L below, determine the maximum allowed width of the transition band of the required analog filter, such as might be used after a zero-order hold (ZOH) interpolator, for: (a) $L = 1$, i.e., a standard D/A; (b) $L = 2$ and (c) $L = 4$

Problem 11

Consider the system illustrated in Fig. 1. The frequency response $H_d(\omega)$ is given by,

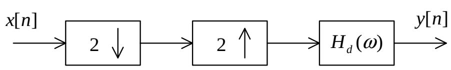

Figure 1: System for Problem 10

$$H_d(\omega) = \begin{cases} 1 & |\omega| \leq \pi/2 \\ 0 & otherwise \end{cases}$$

Find the output $y[n]$ for the following input sequences,

(a) $x[n] = cos(\frac{\pi}{4}n)$

(b) $x[n] = cos(\frac{3\pi}{4}n)$

(c) $x[n] = \frac{sin(\frac{\pi}{8})}{\pi n}$

Problem 12

Given an input $x[n]$, let $v[n]$ be the output of a sample rate expander by an integer factor L, and $w[n]$ be the output of a sample rate compressor by an integer factor D. That is, $w[n] = x[nD]$, and

$$v[n] = \begin{cases} x[k] & n = Lk \\ 0 & otherwise \end{cases}$$

Given that,

$$X_d(\omega) = \begin{cases} 1 - \frac{4|\omega|}{\pi} & |\omega| \leq \pi/4 \\ 0 & \pi/4 \leq |\omega| \leq \pi \end{cases}$$

1. Sketch $V_d(\omega)$ for $L = 2$.

2. Sketch $V_d(\omega)$ for $L = 3$.

3. Sketch $V_d(\omega)$ for $D = 4$.

***Reminder - Homework is due on 08/03/2011 at 5:00 PM - place your assignments in the ECE 410 homework drop box in Everitt Hall!**