University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 311: Digital Signal Processing Lab
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Take Home Exam

Summer 2011

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1 Exam Problems - Due 08/04/2011 at 5:00 PM

1. (30 points) Write a MATLAB function to perform convolution of two sequences using DTFT. The first line of the function should be,

```
function [y] = convTfr(x,h)
```

where `x` and `h` are the sequences to be convolved and `y` is the convolved result.

(a) Use the function to convolve the following sequences,

$$\begin{aligned} x &= [1 \quad 2 \quad -3 \quad 4 \quad -5 \quad 4 \quad -3 \quad 2 \quad 1] \\ h &= [1 \quad 2 \quad -3 \quad 4 \quad -5] \end{aligned}$$

(b) Plot the result using `stem` and verify that it is correct by using `conv`.

Note: The plots must be appropriately labeled and the `x-axis` must show the correct time index.

2. (20 points) Download the file `tones.mat`. This file has 50 samples of a signal consisting of sinusoids at different frequencies.

(a) How many frequency components does the signal have.

(b) Given that the sampling rate $T = \frac{1}{10000}$s what is the value of each frequency component.

3. (20 points) Consider the following signal

$$x(t) = cos(2\pi \times 1000t) + cos(2\pi \times 2000t) + cos(2\pi \times 3000t)$$

Assume a sampling frequency $T = \frac{1}{12000}$.

(a) Compute the DTFT of the given signal and plot the spectrum of the signal. (Choose the fft length so that you can distinguish the frequencies) Note: The plots must be appropriately labeled and show these plots over the range $(0, \pi)$.

(b) Let the component at $\Omega = 2\pi \times 3000$ be the corrupting signal. Design an appropriate filter to filter out this component of the input. You can use `fir1` command for this. Plot the frequency response of this filter.

(c) Filter the input signal $x(t)$ using the designed filter. Compute and plot the DTFT of the filtered signal. Again the plots must be appropriately labeled and the frequency range must be $(0, \pi)$.

4. (40 points) Use the windowing method to design a length-N low-pass, generalized linear phase FIR filter with cut-off frequency $\pi/6$ using both a rectangular window and a hamming window.
Note : The coefficients of a length-N generalized linear phase low pass FIR filter are given by,

$$\{h[n]\}_{n=-\infty}^{\infty} = \frac{\omega_c}{\pi}\text{sinc}\left(\omega_c\left(n - \frac{N-1}{2}\right)\right)$$

where ω_c is the cut-off frequency.
The Hamming window of length N is given by,

$$W[n] = \begin{cases} \left(0.54 - 0.46cos\left(\frac{2\pi n}{N-1}\right)\right), & 0 \le n \le N-1 \\ 0 & , otherwise \end{cases}$$

(a) Write your own function to evaluate the filter's frequency response, $H_d(\omega)$. The function must look as follows:

```
[H w] = freqResp(h,N)
% Function to compute the frequency response of filter h
% Input  : h  - impulse response of the filter
%           N  - number of frequency points at which the response has to be evaluated
%
% Output : H - frequency response of the filter
%          w - frequency vector at which the response was computed
```

(b) Use your Matlab program to evaluate $H_d(\frac{n\pi}{128})$, for $n = -128, -127, \cdots, 127$, for the filters filters with $N = 16$, 32, and 61. For each case, plot their magnitude and phase responses, respectively, and analyze the results to see if they are what you expected.

5. (20 points) Write a MATLAB function to calculate the response of a system characterized by the following difference equation,

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

The input data to the program are the constants a_k, b_k, and the input $x[n]$. Use this function to generate and plot the impulse response of the following system

$$y[n] + 0.7y[n-1] - 0.45y[n-2] - 0.6y[n-3] = 0.8x[n] - 0.44x[n-1] + 0.36x[n-2] + 0.02x[n-3]$$

You can define the input vector to be of length 41 samples and assume zero initial conditions.

Deliverables

- **Reminder: Exam is due on 08/04/2011 at 5:00 PM**