University of Illinois at Urbana-Champaign

ECE 311: Digital Signal Processing Lab

LAB 1: SOLUTIONS

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Problem 1

Parts (a) and (b)

The plots for Parts (a) and (b) are shown in Fig. 1.

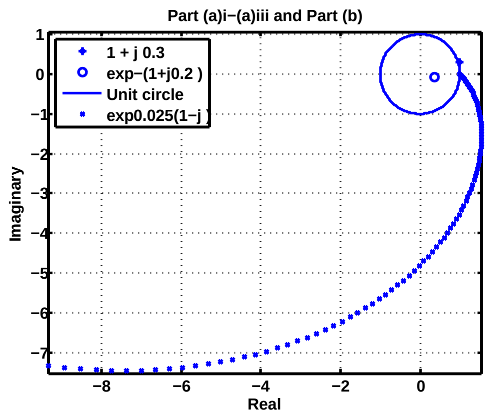

Figure 1: Problem 1. Parts (a) and (b)

Part (c)

The real and imaginary parts are shown in Fig. 2. It can be seen from the figure that both real and imaginary parts decay exponentially.

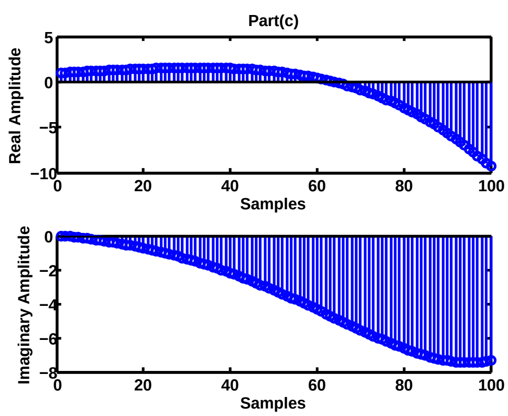

Figure 2: Problem 1. Part (c)

Part (d)

The function f(z) is shown in Fig. 3.

Problem 2

The signal $x_M[n]$ for $M = 4, 5, 7, 10$ is shown in the plots in Figs. 4, 5, 6, and 7 respectively. The fundamental period (N) for each

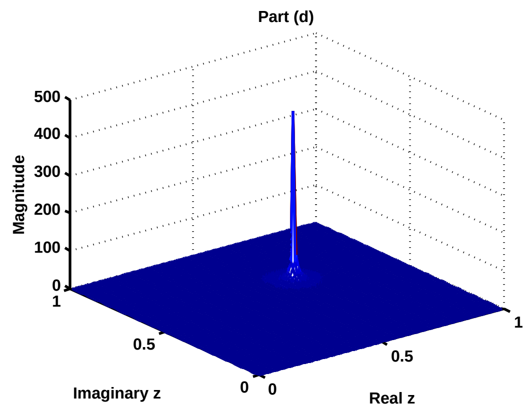

Figure 3: Problem 1. Part (d)

signal is,

$$\begin{aligned}N &= 3(x_4[n]) \\ N &= 12(x_5[n]) \\ N &= 12(x_7[n]) \\ N &= 6(x_10[n])\end{aligned}$$

Consider the condition for periodicity,

$$\begin{aligned}sin(\frac{2\pi M(n+k)}{N}) &= sin(\frac{2\pi Mn}{N}) \\ sin(\frac{2\pi Mn}{N} + \frac{2\pi Mk}{N}) &= sin(\frac{2\pi Mn}{N} + 2\pi k_1)\end{aligned}$$

hence we must have,

$$\frac{kM}{N} = k_1 \quad (1)$$

i.e. $\frac{kM}{N}$ must be an integer.
We first consider $M < N$. Now if M and N are relatively prime then $k = N$ satisfies the condition in (1). If M and N are not relatively prime then express M and N as a product of prime factors,

$$\frac{kM}{N} = \frac{p_1^{a_1}.p_2^{a_2}\dots p_r^{a_r}}{p_1^{b_1}.p_2^{b_2}\dots p_s^{b_s}}$$

canceling common factors,

$$\frac{kM}{N} = \frac{M_1}{N_1} \quad \text{(say)}$$

where M_1 and N_1 are relatively prime. Then $k = N_1$.
Let us take the case of $M = 4$ and $N = 12$. Now $\frac{M}{N} = \frac{4}{12} = \frac{1}{3}$ and hence $k = 3$. For $M = 5$ and $N = 12$ we have $k = 12$. Now consider $M > N$, then if M is a multiple of N then $k = 1$. The case of M not being a multiple of N is similar to the earlier case when M, N were not relatively prime and can be handled similarly.

Problem 3

The signal $x_k[n] = sin(\omega_k n)$ for $k = 1, 2, 4, 6$ is shown in the following Fig. 8. As can be seen there are 3 unique signals. This can be seen easily by writing $sin(w_k n)$ for $k = 6$ as,

$$\begin{aligned}sin(\frac{2\pi 6}{5}) &= sin(\frac{2\pi(5+1)}{5}) \\ &= sin(2\pi + \frac{2\pi}{5}) \\ &= sin(\frac{2\pi}{5})\end{aligned}$$

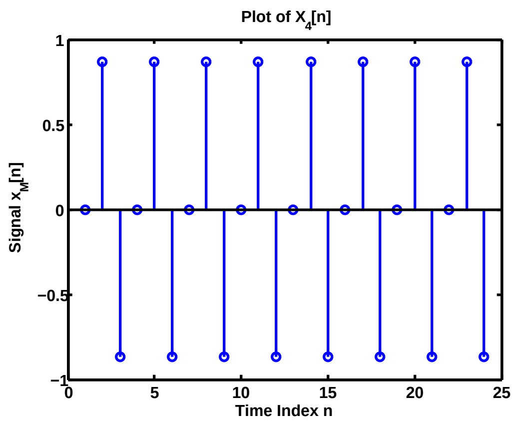

Figure 4: Problem 2. The signal $x_4[n]$

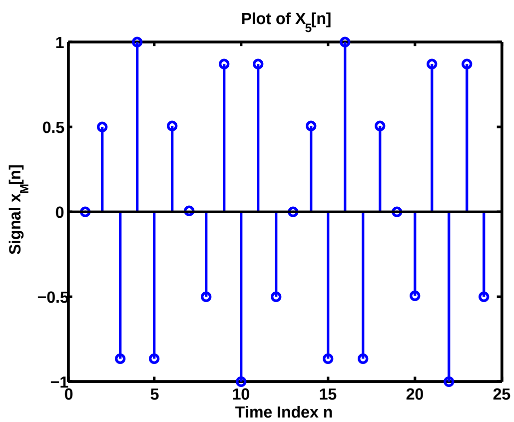

Figure 5: Problem 2. The signal $x_5[n]$

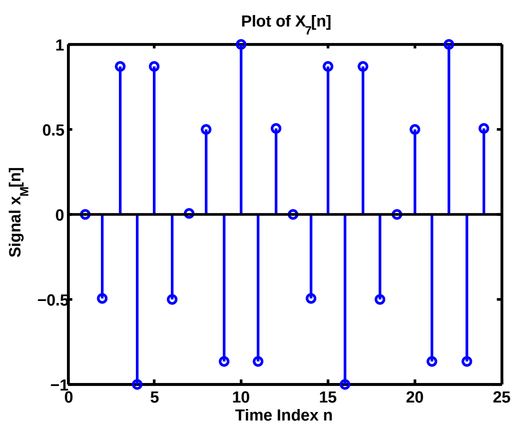

Figure 6: Problem 2. The signal $x_7[n]$

Problem 4

The plots for the three signals, $x_1[n], x_2[n],$ and $x_3[n]$ are shown in Figs. 9, 10, and 11 respectively.

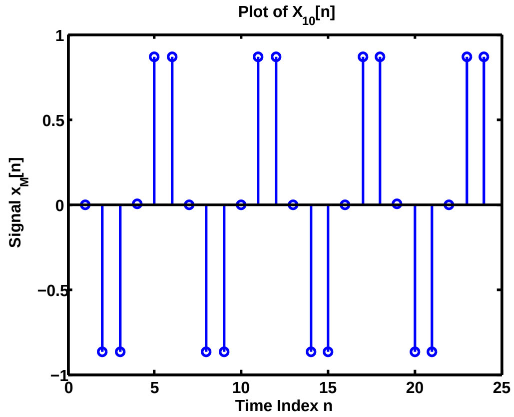

Figure 7: Problem 2. The signal $x_{10}[n]$

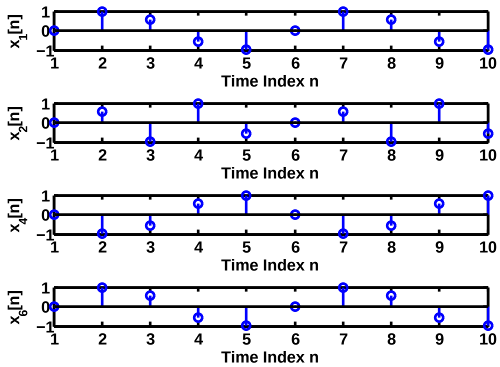

Figure 8: Problem 3. The signal $x_k[n] = sin(w_k n)$ for $k = 1, 2, 4, 6$

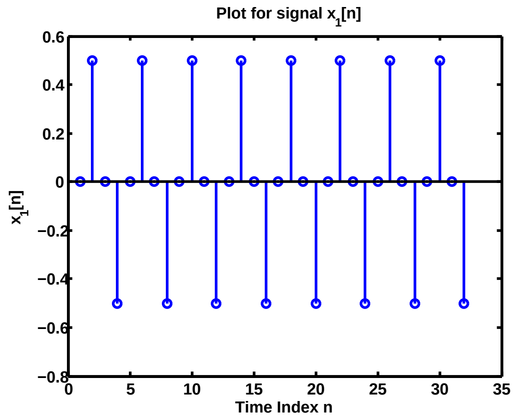

Figure 9: Problem 4. The signal $x_1[n] = sin(\frac{\pi n}{4})cos(\frac{\pi n}{4})$

It is easy to determine the period of the signals without MATLAB. Consider first the signal $x_1[n] = sin(\frac{\pi n}{4})cos(\frac{\pi n}{4})$,

$$\begin{aligned} sin(\frac{\pi n}{4})cos(\frac{\pi n}{4}) &= sin(\frac{\pi n}{4} + \frac{\pi n}{4}) + sin(\frac{\pi n}{4} - \frac{\pi n}{4}) \\ &= sin(\frac{\pi n}{2}) \end{aligned}$$

It can be seen that the fundamental period of $sin(\frac{\pi n}{2})$ is $N = 4$.

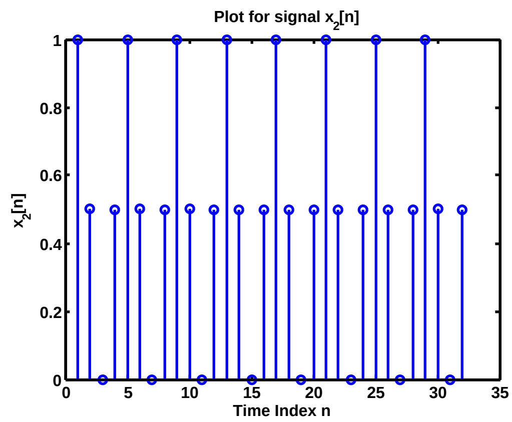

Figure 10: Problem 4. The signal $x_2[n] = cos^2(\frac{\pi n}{4})$

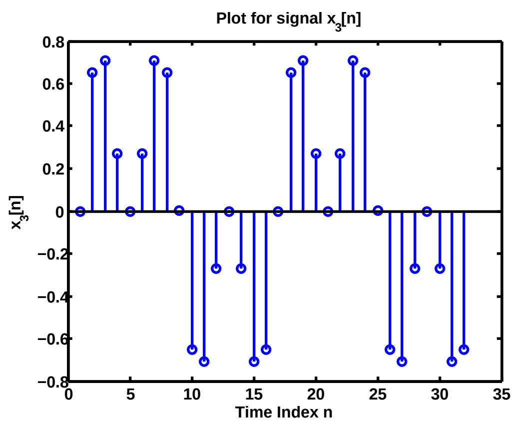

Figure 11: Problem 4. The signal $x_3[n] = sin(\frac{\pi n}{4})cos(\frac{\pi n}{8})$

Similarly $cos^2(\frac{\pi n}{4})$ can be written as,

$$cos^2(\frac{\pi n}{4}) = \frac{1}{2} + \frac{1}{2}cos(\frac{\pi n}{2}) \quad = \quad sin(\frac{\pi n}{2})$$

The fundamental period of $cos(\frac{\pi n}{2})$ is $N = 4$.
Now consider $x_3[n] = sin(\frac{\pi n}{4})cos(\frac{\pi n}{8})$,

$$\begin{aligned} sin(\frac{\pi n}{4})cos(\frac{\pi n}{8}) = sin(\frac{\pi n}{4} + \frac{\pi n}{8}) + sin(\frac{\pi n}{4} - \frac{\pi n}{8}) \\ &= \quad sin(\frac{3\pi n}{8}) + sin(\frac{\pi n}{8}) \end{aligned}$$

Let us first look at $sin(\frac{3\pi n}{8})$. This will be periodic if

$$sin(\frac{3\pi(n+N)}{8}) = sin(\frac{3\pi n}{8})$$

i.e,

$$\frac{3\pi N}{8} = 2\pi k$$

Hence the fundamental period $N_1 = 16$. Now look at $sin(\frac{\pi n}{8})$. This signal is periodic if

$$\frac{\pi N}{8} = 2\pi k$$

Hence the fundamental period signal for $sin(\frac{\pi n}{8})$ is $N_2 = 16$.
Hence the fundamental period of $x_3[n]$ is $N = 16$

Problem 5

The given signal $x[n]$ and the various shifted versions of $x[n]$ denoted by $y_k[n]$ are shown in the Figs. 12 and 13.

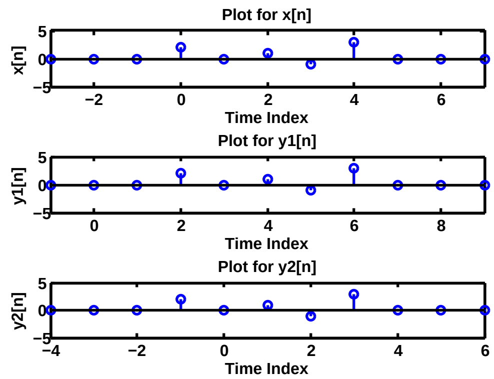

Figure 12: Problem 5. The signal $x[n]$ and its shifted versions $y_1[n], y_2[n]$

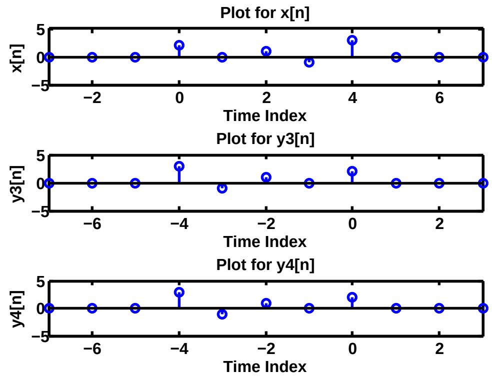

Figure 13: Problem 5. The signal $x[n]$ and its shifted versions $y_3[n], y_4[]$