
LAB 6: FIR Filter Design
Summer 2011

1 Overview

In this lab we will use MATLAB to design FIR filters using truncation, windowing, and frequency sampling.

2 FIR Filter Design

While designing frequency selective filters, we look at the desired filter characteristics in the frequency domain in terms of the desired magnitude and phase response of the filter. In filter design we determine the coefficients of a FIR or IIR filter that closely approximates the desired frequency response specifications. The choice between FIR or an IIR filter depends on the nature of the problem and the desired frequency response. Usually FIR filters are employed in filtering problems where there is a requirement of linear phase characteristic in the passband of the filter. If there is no such requirement one can design either an FIR or an IIR filter. However it must be noted that in general an IIR filter has lower sidelobes in the stopband than an FIR filter having the same number of parameters. In this lab we will focus on FIR filter. FIR filters are described by the following difference equation,

$$y[n] = \sum_{m=0}^M b[m]x[n-m]$$

where $y[n]$ is the filtered signal, $b[m]$ are the numerator coefficients of the transfer function.

3 Ideal and non-ideal filter characteristics

Consider the impulse $h(n)$ of an ideal lowpass filter with frequency response characteristic

$$H(\omega) = \begin{cases} 1, & |\omega| \leq |\omega_c| \\ 0, & |\omega_c| \leq \omega \leq \pi \end{cases}$$

The impulse response of this filter is shown below,

$$h[n] = \begin{cases} \frac{\omega_c}{\pi}, & n = 0 \\ \frac{\omega_c}{\pi} \frac{\text{sinc}(\omega_c n)}{\omega_c n}, & n \neq 0 \end{cases}$$

Consider the above expression for $\omega_c = \frac{\pi}{4}$. Is the impulse response physically realizable. How will you will plot this function in MATLAB?

We will truncate this function to the interval $(-100, 100)$. Now plot the truncated $\text{sinc}(\cdot)$ function.

```
n = [-100:100]
x = (1/4)*n;
plot(n,sinc(x));
```

Is the truncated impulse response physically realizable?

3.1 Causality

It is clear that the ideal low pass filter and its truncated version are not causal and cannot be realized in practice. One possible solution is to introduce a large delay n_0 in $h[n]$ and arbitrarily to set $h[n] = 0$ for $n < n_0$. This makes the filter causal. Causality results in important implications in filter design,

- The frequency response of $H(\omega)$ cannot be zero except at a finite set of points in frequency.
- Magnitude $|H(\omega)|$ cannot be constant in any finite range of frequencies and the transition from pass-band to stop-band cannot be infinitely sharp.
- The real and imaginary parts of $|H(\omega)|$ are interdependent and hence the $|H(\omega)|$ and $\angle H(\omega)$ cannot be arbitrarily chosen.

The realizable magnitude response is shown in Fig. 1.

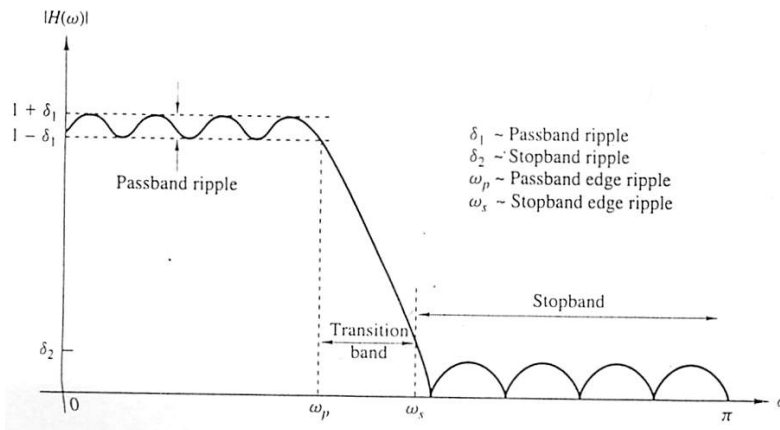


Figure 1: Realizable Magnitude response

4 Linear Phase Filters

A filter whose impulse response is symmetric or antisymmetric about its midpoint is called a (generalized) linear phase filter. That is,

$$h[n] \longleftrightarrow H(\omega) = |H(\omega)|e^{jk\omega}$$

For such filters the phase of the filtered signal will vary linearly with frequency ω . The phase delay ($k\omega/\omega$) and ($\frac{d}{d\omega}$) will be equal and constant. What is the phase and group delay for an order n linear phase FIR filter? Consider the following FIR filter,

$$y[n] = \frac{1}{2} (x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]);$$

Try the following

```
h = 0.2*ones(1,5)
freqz(h);
```

Is the FIR filter linear phase?

A system is referred to as generalized linear-phase system if its frequency response can be expressed in the form,

$$H_d(\omega) = |H_d(\omega)|e^{-j\alpha+j\beta}$$

where α and β are constants. The symmetric impulse response of a linear phase filter can have an odd or even number of points, and can have an odd or even symmetry about the midpoint, leading to four filter types:

1. *Type I* :

- Even Symmetry : $h[n] = h[M - n], 0 \leq n \leq M$
- M is an even integer

2. *Type II* :

- Even Symmetry : $h[n] = h[M - n], 0 \leq n \leq M$
- M is an odd integer
- $H_d(\pi) = 0$

3. *Type III* :

- Odd Symmetry : $h[n] = -h[M - n], 0 \leq n \leq M$
- M is an even integer
- $H_d(0) = 0$
- $H_d(\pi) = 0$

4. *Type IV* :

- Even Symmetry : $h[n] = -h[M - n], 0 \leq n \leq M$
- M is an odd integer
- $H_d(0) = 0$

4.1 FIR filter design by windowing

In this method we begin with desired frequency response specification $H_d(\omega)$ and determine the corresponding unit sample response $h_d[n]$. $|H_d(\omega)|$ and $h_d[n]$ are related by the following Fourier transform relation,

$$H_d(\omega) = \sum_{n=0}^{\infty} h_d[n]e^{-j\omega n} \quad (1)$$

$$h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega)e^{j\omega n} d\omega \quad (2)$$

As mentioned earlier, the unit sample response $h_d[n]$ obtained from (2) is infinite in duration and must be truncated at some point, say $n = M - 1$, to yield a FIR filter of length M . Truncation of $h_d[n]$ to a length $M - 1$ is equivalent to multiplying $h_d[n]$ by a 'rectangular window' defined as,

$$w[n] = \begin{cases} 1, & n = 0, 1, \dots, M - 1 \\ 0, & \text{otherwise} \end{cases}$$

The Fourier transform of the rectangular window is,

$$W(\omega) = e^{-j\frac{\omega(M-1)}{2}} \operatorname{sinc} \frac{\omega M}{2}$$

Plot on the same figure the magnitude and phase response of the window function for window lengths $M = 31$ and $M = 121$. What happens to the main lobe? What happens to the side-lobes? What can you say about the phase response?

The characteristics of the rectangular window play a significant role in determining the resulting frequency response of the FIR filter obtained by truncating $h_d[n]$ to length M . Recall applying a window function in the time domain is equivalent to convolution in the frequency domain and has the effect of smoothing $H_d(\omega)$. As M is increased, $W(\omega)$ becomes narrower and the smoothing action of $W(\omega)$ is reduced. On the other hand large side-lobes of $W(\omega)$ has the effect of undesirable ringing effects. The undesirable ringing effects in FIR filters can be reduced by using windows that do not have abrupt discontinuities in their time-domain characteristics. Table 1 lists some window functions, Using

Table 1: Window Functions for FIR filter design

Window	Time Domain Sequence
Hamming	$0.54 - 0.46 \cos \frac{2\pi n}{M-1}$
Blackman	$0.42 - 0.5 \cos \frac{2\pi n}{M-1} + 0.08 \cos \frac{4\pi n}{M-1}$
Hanning	$\frac{1}{2} \left(1 - \cos \frac{2\pi n}{M-1} \right)$
Kaiser	$\frac{I_0 \left[\alpha \sqrt{\left(\frac{M-1}{2} \right)^2 - (n - \text{frac}M - 12)^2} \right]}{I_0 \left[\alpha \left(\frac{M-1}{2} \right) \right]}$

MATLAB plot the magnitude and phase response of the Hamming, Blackman and Hanning windows. Use Window length of $N = 31, 121$. Compare your results to those obtained by rectangular window. Matlab has several window functions. You can see them by typing `help window`. Matlab also has several functions to design filters. The function `fir1` can be used to design a M^{th} order FIR digital filter. Try the following

```
help fir1
h = fir1(31,1/2,'low');
freqz(h);
h = fir1(31,1/2,'high');
freqz(h);
h = fir1(31,1/2,hamming(32));
freqz(h);
```

Some other filter design functions are, `fir2`, `firpm`, `fircls1`, `firls`.

4.2 Filter Design by Frequency Sampling

In frequency sampling method of FIT filter design, we specify the desired frequency response $H_d(\omega)$ at a set of equally spaced frequencies, namely,

$$H\left(\frac{2\pi k}{15}\right) = \begin{cases} 1, & k = 0, 1, 2, 3 \\ 0.4, & k = 4 \\ 0, & k = 5, 6, 7 \end{cases}$$

We then solve for unit sample response $h_d[n]$ of the filter from these equally spaced specifications. To reduce side-lobes, it is desirable to optimize the frequency specification in the transition band of the filter. The optimization can be done by on computer using linear programming techniques.

5 Homework - Due 07/26/2011 at 5:00 PM

1. You desire to design a generalized linear phase (GLP) FIR low-pass filter with $\omega_c = \pi/4$ using a truncation (windowing) method. Beginning with an ideal LPF $h_{LPF}[n] = \frac{\sin(\frac{\pi}{4}n)}{\pi n}$, we can design a FIR filter by shifting $h_{LPF}[n]$ by $\frac{N-1}{2}$ and multiplying by window function $w'[n]$,

$$h[n] = \frac{\sin\left(\frac{\pi}{4}\left(n - \frac{N-1}{2}\right)\right)}{\pi\left(n - \frac{N-1}{2}\right)} w'[n], \quad n = 0, 1, \dots, N-1.$$

- (a) For a rectangular window with length $N = 21$, plot $h[n]$, $|H_d(\omega)|$, and $\angle H_d(\omega)$ in MATLAB. Check the cut-off frequency of this filter from the graph of $|H_d(\omega)|$. Use 4096-point DFT to generate $|H_d(\omega)|$ and $\angle H_d(\omega)$. Show these plots over the range of $[0, \pi]$.
 - (b) Consider a hamming window of with length $N = 21$, i.e., $w'[n] = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$. Do the same thing as in (b) and compare the results.
 - (c) Consider rectangular window with length $N = 121$. Do the same things as in (b) and compare the results.
2. You desire to design a 121- tap generalized linear phase (GLP) FIR low-pass filter with $\omega_c = \frac{\pi}{4}$ using frequency sampling method.
 - (a) Write down the sampled frequency response $H_d(\omega)$ at $\omega = \frac{2\pi k}{N}$ for $0 \leq k \leq N-1$ (hint: consider generalized linear phase)
 - (b) Determine the impulse response of this LPF using inverse-fft in MATLAB (`ifft`). Plot $h[n]$, $|H_d(\omega)|$ and $\angle H_d(\omega)$ in MATLAB. Use a 4096 -point fft to generate $|H_d(\omega)|$ and $\angle H_d(\omega)$. Show the plots over a range of $[0, \pi]$.
 - (c) Construct a high-pass filter (HPF) from this filter using modulation property. Plot $h[n]$, $|H_d(\omega)|$, and $\angle H(\omega)$ of this HPF with MATLAB.
 3. The expression for the coefficients $\{h_n\}_0^{31}$ of a length-32 FIR low-pass filter having cutoff at $w_c = \frac{\pi}{4}$ radians using the window design method with Hamming window is given by,

$$\{h_n\}_0^{31} = \frac{1}{4} \text{sinc} \left[\frac{\pi}{4} \left(\frac{n-31}{2} \right) \right] \left[0.54 - 0.46 \cos \left(\frac{2\pi n}{31} \right) \right] \quad (3)$$

- (a) Design the same filter using MATLAB command `fir1`.
 - (b) Plot both the magnitude and phase of the frequency response. Comment on the plots in relation to the frequency response of an ideal low-pass filter. What are the pass-band ripple and stop band attenuations of your filter in dB? What is the transition bandwidth?
 - (c) Generate an input signal consisting of sum of two sinusoids with $\omega = \frac{\pi}{6}$ and $\omega = \frac{\pi}{3}$. Convolve this signal with the filter specified in (3). Explain the results based on frequency response of the filter.
4. This problem requires you to work with three different low-pass filters which are specified by the coefficient vectors stored in the file `phdist.mat`, on the course website. Load the the file using the `load` command. If this file has been loaded correctly, then typing `who` should result in

```
>>who
Your variables are:
a1  a3  b2  xnoise
a2  b1  b3
```

The vectors `a1` through `a3` and `b1` through `b3` contain the coefficient vectors for three discrete-time filters in the format required by `freqz` and `filter`. The vector `xnoise` contains the speech signal with some high-frequency noise added. The original speech signal is in the file `orig.mat` on the course website. Once you load the `orig.mat` file the uncorrupted version of the speech signal will be in the vector `x`.

- (a) Use `freqz` to compute the frequency response of all the three filters at 1024 evenly spaced points on the interval $0 \leq \omega \leq \pi$. Plot the frequency response magnitude of all three filters. Suppose you used the same speech signal as input to all three filters, based on the magnitudes of the frequency responses, which two of the output signals would you expect to sound most similar?
- (b) Generate appropriately labeled plots of the phase of the frequency response for all three systems. The function `angle` will return angles between $-\pi$ and π . If just plot the output of `angle`, you may find it hard to understand some of the plots, since phase will "wrap around" from the top to the bottom of the graph every time it passes π and $-\pi$. The function `unwrap` takes the output of `angle` and removes the "wrap around" effect to obtain a more continuous phase function. For example, if `H` contains the samples of the frequency response at frequencies specified in `omega` then plot `plot(omega/pi, unwrap(angle(H)))` displays the unwrapped phase. Which two filters are most alike in phase response?
- (c) Define the vectors `y1` through `y3` to be the output of the filters when they are applied to the clean speech signal in `x` using `filter`. Play each speech signal using `soundsc`. Is the speech still easy to understand in each case? Which two speech signals sound most alike? Based on your results for this input signal, would you say that the magnitude or phase of these filters is more important?
- (d) Play both `x` and `xnoise` using `sound` to hear the difference between clean and corrupted speech signals.
- (e) Use `fft` to compute the DTFT of `x` and `xnoise` at 8192 evenly spaced samples in ω and store these values `X` and `Xn`. Plot the magnitude of the DTFT for each signal. Can you see the high frequency noise in the DTFT of the corrupted signal?
- (f) Let `yn1` and `yn3` be the result of filtering `xnoise` with each of the three filters. Listen to all three of these signals. Which filter best eliminated the noise?
- (g) Use `fft` to compute the DTFT of the filtered speech signals at 8192 evenly spaced points in ω and store them in `Yn1` through `Yn3`. Make appropriately labeled plots of the magnitude of the DTFT for each signal. Comment on your results.

Deliverables

- Email your code, figures, calculation and answers as a *.pdf* or *.doc* file to ece311lab.uiuc@gmail.com. Be sure to name your document in the form- **ECE311Lab6_firstname_lastname.doc/pdf**.
- Late reports will reduce the grade by 20% per day.
- Make sure to present a clear and concise report having figures labeled and centered.
- **Reminder: Homework is due on 07/26/2011**