University of Illinois at Urbana-Champaign
Department of Electrical and Computer Engineering

ECE 311: Digital Signal Processing Lab
Chandra Radhakrishnan
Peter Kairouz

LAB 7: **IIR Filter Design**
Summer 2011

---

1 Overview

In this lab we will use MATLAB to design IIR filters using Bilinear Transforms

2 IIR Filters

While designing frequency selective filters, we look at the desired filter characteristics in the frequency domain in terms of the desired magnitude and phase response of the filter. In filter design we determine the coefficients of a FIR or IIR filter that closely approximates the desired frequency response specifications. The main advantage of IIR filters over FIR filers is that they can meet a given set of specifications with a much lower filter order compared to an FIR filter designed to meet the same specifications. It should be noted that IIR filters have nonlinear phase and one may have to do further processing to overcome the effects of nonlinear phase on the filtered data. This can done if one has access to entire data before the filtering operation is done, i.e. the processing is not done in real time. IIR filters can be represented as,

$$y[n] = \sum_{k=1}^{L} a[k]y[n-k] + \sum_{m=0}^{M} b[m]x[n-m]$$

where $y[n]$ is the filtered signal, $b[m]$ are the numerator coefficients of the transfer function.

3 IIR filter types

In this course we look at three classical IIR filters - Butterworth, Chebyshev, and Elliptic filters. All these filters approximate the ideal ''brick wall' filter. MATLAB provides functions to create all the three type of filters in both analog and digital domains. We can also specify the frequency response characteristics of the filters in terms of whether the filters are lowpass, highpass, bandpass, or bandstop. Table 1 lists the magnitude response function, properties and parameters of Butterworth, Chebyshev, and Elliptic filters.

4 IIR Filter design by Analog Prototyping

The standard approach to designing discrete-time IIR filters is the following:

1. Estimate filter order.

Table 1: IIR Filter Types

Filter Type	Magnitude Response $M(\Omega^2)$
Butterworth	$\dfrac{1}{1+(\frac{\Omega}{\Omega_c})^{2N}}$
Chebyshev	$\dfrac{1}{1+\epsilon^2 C_N^2\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$
Elliptic	$\dfrac{1}{1+\epsilon^2 J_n^2(\Omega)}$

2. Design a prototype analog lowpass filter $(H_L(s))$ with normalized cut-off frequency 1.

3. Translate the filter to the desired band configuration.

4. Discretize the filter i.e. convert $H_L(s)$ to $H(z)$

In filter design usually the magnitude specifications are given with some tolerances. Fig. 1 shows a typical magnitude response specification of an analog filter. The magnitude response specification in a normalized form can also be given. These specifications are shown in Fig. 2.
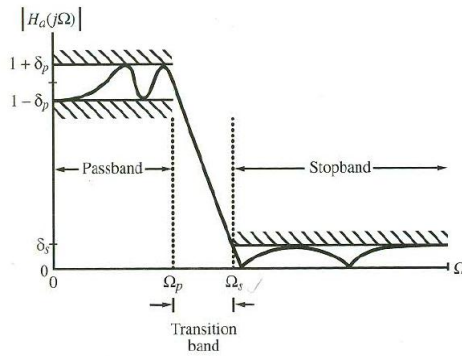

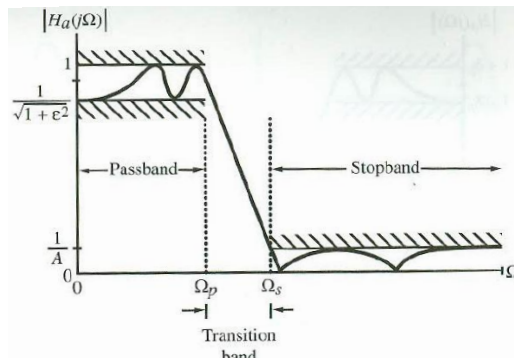Figure 1: Magnitude response specification of a lowpass filter

Figure 2: Normalized magnitude response specification of a lowpass filter

2

4.1 Filter Order Estimation

In the design of IIR digital lowpass filter $H(z)$ based on the conversion of an analog lowpass filter $H_a(s)$, the filter order of $H_a(s)$ is first estimated from its specifications using the formulas given below: where ρ for the Elliptic filter in Table

Table 2: Filter Order Estimation

Filter Type	Order Estimation Formula
Butterworth	$N = \frac{1}{2}\dfrac{log_{10}\left[(A^2-1)/\epsilon^2\right]}{log_{10}\left(\frac{\Omega_s}{\Omega_p}\right)}$
Type 1 Chebyshev	$N = \dfrac{cosh^{-1}\left[(\sqrt{A^2-1})/\epsilon\right]}{cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$
Elliptic	$N = \dfrac{2log_{10}\left(\frac{\sqrt{A^2-1}}{\epsilon}\right)}{log_{10}(\frac{1}{\rho})}$

2 is defined as follows,

$$k = \sqrt{1-\left(\frac{\Omega_p}{\Omega_s}\right)^2}$$

$$\rho_0 = \frac{1-\sqrt{k}}{2(1+\sqrt{k})}$$

$$\rho = \rho_0 + 2(\rho_0)^5 + 15(\rho_0)^9 + 150(\rho_0)^{13}$$

MATLAB provides the functions `buttord`, `cheb1ord`, `cheb2ord`, and `ellipord` to compute the order of the filters. The function inputs are

```
Wp = passband edge (rad/s)
Ws = stopband edge (rad/s)
Rp = maximum passband attenuation (dB)
Rs = minimum stopband attenuation (dB)
's' = parameter to design an analog filter
```

The function returns,

```
N = The filter order
Wn = Angular cut-off frequency frequency
```

Let us now estimate the filter order for an analog Butterworth lowpass filter using `butterord`. The specifications are Wp = 2\pi(1000), Ws=2\pi(5000),Rp=1, and Rs=40

```
[N, Wn] =  buttord(Wp,Ws,Rp,Rs,'s')
N =
     4
Wn =
  9.9347e+003
```

Try using the commands for the remaining filter types.

4.2 Analog system function design

Once the filter order and the cut-off frequency is estimated we need to find the system function from the magnitude response expression (Table 1)for the chosen filter (Butterworth, Chebyshev, Elliptic). This is done by factorizing $M(-s^2)$ as follows,

$$M(-s^2) = H_a(s)H_a(-s).$$

We then take $H_a(s)$ to be the left half plane pole factors and left half plane zero factors. MATLAB provides functions for carrying out this. MATLAB functions are shown below:

Butterworth

```
[z,p,k]   = buttap(N)
[num,den] = butter(N,Wn,'s')
[num,den] = butter(N,Wn,'type','s')
```

Chebyshev I

```
[z,p,k]   = cheb1ap(N,Rp)
[num,den] = cheby1(N,Rp,Wn,'s')
[num,den] = cheby1(N,Rp,Wn,'type','s')
```

Chebyshev II

```
[z,p,k]   = cheb2ap(N,Rp)
[num,den] = cheby2(N,Rp,Wn,'s')
[num,den] = cheby2(N,Rp,Wn,'type','s')
```

Elliptic

```
[z,p,k]   = ellipap(N,Rp,Rs)
[num,den] = ellipap(N,Rp,Rs,Wn,'s')
[num,den] = ellip(N,Rp,Wn,Ws,'type','s')
```

Use `help` to see what each function does. In this lab we will use `buttap`, `cheby1ap`, and `ellipap` to design filters. These functions return the poles and zeros of the continuous time normalized transfer function. We can obtain the normalized transfer function using the MATLAB function `zp2tf`. We can then ''un-normalize' using `lp2lp`.

4.3 Bilinear Transformation

The designed analog prototype system function $H_a(s)$ must now be converted to a digital transfer function $H(z)$. This is done by using Bilinear Transformation,

$$H(z) = H_a(s)|_{s=\alpha\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

MATLAB provides the function `bilinear` to implement this operation. Remember that bilinear transformation introduces *frequency warping*. We must *prewarp* the frequencies before we design our analog prototype.

5   Homework - Due 08/02/2011 at 5:00 PM

1. Design a digital Butterworth lowpass filter operating at a sampling rate of 80 kHz with a 0.5-dB cutoff frequency at 4 kHz and a minimum stopband attenuation of 45 dB at 20 kHz using bilinear transformation method. Determine the order of the analog filter prototype using the `buttord` function. Then design the filter using `buttap`. Use `zp2tf` to convert the poles and zeros you obtain to a transfer function and then use `lp2lp` to transform the filter to desired cut-off. Plot the magnitude and phase response of the designed filter.

2. Design a Type I Chebyshev lowpass filter operating at a sampling rate of 80 kHz with a 0.5-dB cutoff frequency at 4 kHz and a minimum stopband attenuation of 45 dB at 20 kHz using bilinear transformation method. Design the analog prototype using `cheb1ap`. Use `zp2tf` to convert the poles and zeros you obtain to a transfer function and then use `lp2lp` to transform the filter to desired cut-off. Get the discrete-time filter using `bilinear` Plot the magnitude and phase response of the designed filter.

3. Design a Elliptic lowpass filter operating at a sampling rate of 80 kHz with a 0.5-dB cutoff frequency at 4 kHz and a minimum stopband attenuation of 45 dB at 20 kHz using bilinear transformation method. Design the analog prototype using `ellipap`. Use `zp2tf` to convert the poles and zeros you obtain to a transfer function and then use `lp2lp` to transform the filter to desired cut-off. Get the discrete-time filter using `bilinear`. Plot the magnitude and phase response of the designed filter.

Deliverables

- Email your code, figures, calculation and answers as a *.pdf* or *.doc* file to **ece311lab.uiuc@gmail.com**. Be sure to name your document in the form- **ECE311Lab6_firstname_lastname.doc/pdf**.

- Late reports will reduce the grade by 20% per day.

- Make sure to present a clear and concise report having figures labeled and centered.

- **Reminder: Homework is due on 08/02/2011**