

# INTERFERENCE-AWARE RATE CONTROL FOR BURSTY INTERFERENCE CHANNELS

Peter Kairouz, Ahmed K. Sadek, Tamer Kadous

Qualcomm Research

San Diego, California 92121

Email: kairouz2@illinois.edu and {asadek,tkadous}@qti.qualcomm.com

## ABSTRACT

Interference in wireless networks has been identified as one of the main hurdles towards achieving higher network capacity. However, most of the literature has focused on solving interference problems assuming that interference is non-bursty. In this paper, we study bursty interference channels and propose novel interference-aware rate control algorithms. The proposed algorithms include single and multi-layer transmission schemes. We also present a framework for optimizing rate selection so that the overall throughput is maximized. Significant performance gains relative to traditional Hybrid Adaptive Repeat reQuest (HARQ) schemes are demonstrated.

**Index Terms**— Interference channels, bursty interference, rate control, HARQ, broadcast channels, superposition coding, optimization.

## 1. INTRODUCTION

In this paper, we develop new transmission and rate selection algorithms for interference channels. Our interference-aware rate control algorithms apply to a broad class of interference channels such as bursty interference channels, broadcast channels, multiple-access (MAC) interference channels, and inter-cell interference channels. However, particular attention will be given to bursty interference channels. Cross device interference, e.g. a Wi-Fi device interfering with an LTE device, is one example of bursty interference that can emerge in scenarios where LTE and WiFi are using adjacent channels with insufficient guard band (e.g. B40 and ISM band). We propose various transmission schemes and apply rate selection algorithms that are designed to maximize the overall throughput. This leads to a significant improvement in performance when compared to existing methods.

**Relation to prior work:** Recently, automatic rate control using rateless coding has been suggested for MAC interference channels [1, 2]. However, their schemes assume that the receiver can decode the interfering signals and subtract them out from the received block. This assumption is not valid for bursty interference channels where the jammer's modulation and coding schemes are unknown to the receiver. Therefore, we take a different route to interference management and rate

control. This paper extends the traditional concept of HARQ [3, 4] to multi-layer HARQ and introduces a framework for optimizing the rate selection.

## 2. SYSTEM MODEL

An  $N$ -jammer bursty interference channel is described by

$$\mathbf{y}_m = \mathbf{x}_m + \sum_{j=1}^N b_{j,m} \mathbf{w}_{j,m} + \mathbf{v}_m \quad (1)$$

where  $\mathbf{x}_m$  is a vector of  $L$  symbols transmitted over the  $m^{\text{th}}$  slot,  $\mathbf{v}_m$  is thermal noise,  $\mathbf{w}_{j,m}$  is an interference vector due to a jammer, and  $b_{j,m}$  is a Bernoulli random variable with mean  $\alpha_j$  representing the  $j^{\text{th}}$  jammer duty cycle. The vectors  $\mathbf{v}_m$  and  $\mathbf{w}_{j,m}$  are assumed to have independent and identically distributed (i.i.d.) zero-mean complex Gaussian entries with variances  $N_0$  and  $I_j$  respectively. The receiver is only interested in decoding  $\mathbf{x}_m$ . The decoding of  $\mathbf{w}_{j,m}$ 's is not possible because the receiver is oblivious to the modulation and coding schemes used by the interferers. The instantaneous capacity of (1) is a discrete random variable with up to  $2^N$  possible outcomes and its average is given by

$$\mathbb{E}[C(\boldsymbol{\alpha})] = \sum_{\mathbf{b} \in \{0,1\}^N} \left( \prod_{j=1}^N \alpha_j^{b_j} (1 - \alpha_j)^{1-b_j} \right) \times \log_2 \left( 1 + \frac{P}{\sum_j b_j I_j + N_0} \right) \quad (2)$$

where  $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_N)$  represents the vector of duty cycles and  $\mathbf{b} = (b_1, \dots, b_N) \in \{0,1\}^N$ . As a special case, when  $N=1$ , the channel capacity simplifies to

$$C = \begin{cases} C_g = \log_2(1 + \gamma_g), & \text{w.p. } 1 - \alpha \\ C_b = \log_2(1 + \gamma_b), & \text{w.p. } \alpha \end{cases} \quad (3)$$

where  $\gamma_g$  represents the SNR= $P/N_0$  and  $\gamma_b$  represents the SINR= $P/(I + N_0)$ . Here,  $C$  is a binary random variable with an expected value  $\mathbb{E}[C] = \alpha C_b + (1 - \alpha) C_g$ . The transmitter does not have prior information on when the interference collides with a transmitted packet. However, we assume that

the transmitter has an estimate of the interference statistics  $\mathcal{P} = \{\alpha, \mathbf{I}, N_0\}$ , where  $\mathbf{I} = (I_1, \dots, I_N)$ .

### 3. SINGLE-LAYER SCHEMES

In this section, we review and extend single-layer schemes: a single codebook is used to encode and transmit codewords over an interference channel. These schemes are divided into two categories: single-transmission and re-transmission methods. The HARQ protocol with a maximum of  $M$  re-transmissions is a popular re-transmission method [3, 4]. Single-transmission methods are a special case of re-transmission methods with  $M=1$ . We now define the HARQ MTBRS rate selection algorithm.

**Definition 3.1 (HARQ MTBRS)** *The maximum throughput based rate selection (MTBRS) algorithm solves for*

$$R^* = \operatorname{argmax} T(R, \mathcal{P}, M) \quad (4)$$

Observe that the throughput is a function of the rate  $R$ , interference statistics  $\mathcal{P}$ , and the maximum number of re-transmissions  $M$ .

#### 3.1. Single-Transmission Methods

In single-transmission methods (STMs), the transmitter uses a codebook  $\mathcal{C} \subset \mathbb{C}^L$  with transmission rate  $R = n/L$  to encode a message of  $n$  bits into  $L$  complex symbols. For every rate  $R$ , an associated outage probability  $\delta(R)$  can be computed for a given statistical interference model. An outage event is defined as the event when  $R \geq C$ , and hence  $\delta(R) = P(R \geq C)$ . For STMs, the throughput of the system is given by  $T(R) = R(1 - \delta(R))$ . Having defined  $T(R)$  and  $\delta(R)$ , the task of the transmitter is to solve the rate selection problem defined in (4).

**Theorem 3.2 (1-jammer channel)** *For the 1-jammer channel, the solution to MTBRS is given by*

$$R^* = \begin{cases} C_g & \text{for } \alpha \leq \alpha^* = 1 - \frac{C_b}{C_g} \\ C_b & \text{for } \alpha > \alpha^* \end{cases} \quad (5)$$

and the throughput achieved is  $(1 - \alpha)C_g$  for  $\alpha \leq \alpha^*$  and  $C_b$  for  $\alpha > \alpha^*$ . If, in addition to maximizing the throughput, a maximum outage constraint  $P(R \geq C) \leq \delta$  is imposed, the solution is given by

$$R^* = \begin{cases} C_g & \text{for } \alpha < \delta \\ C_b & \text{for } \alpha \geq \delta \end{cases} \quad (6)$$

and the throughput achieved is  $(1 - \alpha)C_g$  for  $\alpha < \delta$  and  $C_b$  for  $\alpha \geq \delta$ .

**Proof 3.3** *The rate selection optimization problems are straightforward for this case.*

We call this method *threshold based rate selection* (TBRS) because it amounts to comparing the duty cycle  $\alpha$  to a threshold and choosing  $R$  accordingly. We note that  $\alpha^*$  depends on  $C_g$  and  $C_b$  through their ratio. In the sequel, this will turn out to be generally true for HARQ MTBRS schemes when applied to bursty interference channels.

#### 3.2. Re-transmission Methods

In re-transmission methods, the transmitter encodes  $n$  information bits using a codebook  $\mathcal{C} \subset \mathbb{C}^{LM}$  of size  $LM$  symbols. The codewords are divided into  $M$  sub-blocks, each of length  $L$ . The transmitter sends one block at a time until either an acknowledgment (ACK) is received or all  $M$  re-transmissions are exhausted. The per sub-block rate of transmission is  $R = n/L$  and the effective rate after  $m$  transmissions is equal to  $R/m$ . The asymptotic performance of this scheme for MAC channels has been studied in [4]. We investigate its performance for bursty interference channels. As mentioned in Section 2, the capacity of (1) is a discrete random variable with up to  $2^N$  outcomes. By applying the renewal-reward theorem [5], the throughput of HARQ systems is given by

$$T(R) = \frac{(1 - \delta(R))R}{\bar{m}(R)} \quad (7)$$

$$\delta(R) = P\left(R \geq \sum_{i=1}^M C_i\right)$$

$$\begin{aligned} \bar{m}(R) = P(R < C_1) &+ \sum_{i=2}^{M-1} iP \left( \sum_{j=1}^{i-1} C_j \leq R < \sum_{j=1}^i C_j \right) \\ &+ MP \left( R \geq \sum_{i=1}^{M-1} C_i \right) \end{aligned}$$

where  $C_i$ ,  $\delta(R)$ , and  $\bar{m}(R)$  denote the capacity of the channel in slot  $i$ , outage probability, and average number of re-transmissions respectively. Assuming that the interference is independent across transmission slots, the pmf of  $\sum_{i=1}^M C_i$  is given by the  $M$ -fold convolution of  $C_i$ 's pmf. For a given interference channel and a fixed value of  $M$ , one can use (7) to solve for  $R^*$ . As  $M \rightarrow \infty$ , if we let  $R^* = M\mathbb{E}[C]$ , then  $\delta(R^*) \rightarrow 0$  by the weak law of large numbers. In addition,  $T^* \rightarrow \mathbb{E}[C]$ , and hence HARQ is asymptotically optimal for any interference channel. This comes at the expense of increased latency and decoding complexity.

**Proposition 3.4 (1-jammer HARQ MTBRS)** *For the 1-jammer channel,  $R^*$  can take one of only  $(M^2 + 3M)/2$  values for any  $C_g, C_b, \alpha$ , and  $M$ . In addition, let  $C_g/C_b = \epsilon$ , the choice of  $R^*$  depends on  $C_g$  and  $C_b$  only through their ratio  $\epsilon$ .*

**Proof 3.5** *Omitted for space limitations.*

The above proposition shows that for a given  $\epsilon$ ,  $\alpha$ , and  $M$ , we can solve for  $R^*$  in a very efficient way. For example, for  $M = 2$ ,  $\{C_b, C_g, 2C_b, C_b + C_g, 2C_g\}$  are the only rates that need to be checked.

#### 4. MULTI-LAYER SCHEMES

In this section, we propose using Superposition Coding (SPC) combined with rate selection to further improve the system's throughput. SPC was first introduced in [6] as an optimal transmission strategy for broadcast channels: a single node communicating with multiple nodes. We apply this strategy for bursty interference channels. Even if the transmitter cannot predict the interference levels a priori, it can target the "good" and "bad" cases simultaneously by transmitting two (or more) codewords: a high rate one and a low rate one. When there is no interference both codewords can be jointly decoded. However, when interference is present, we can still decode the low rate codeword. Finally, in addition to using SPC, we present a hybrid scheme that combines the benefits of both HARQ and SPC.

##### 4.1. Superposition Coding Methods

In this approach, the transmitter uses multiple independent codebooks  $C_k$ ,  $k \in \{1, \dots, M\}$ , with different rates  $R_k$ . In every transmission slot,  $M$  codewords (layers) are chosen, scaled by  $\sqrt{\eta_i P}$  each, linearly combined, and transmitted simultaneously over the channel [6, 7]. Here,  $\eta_i$  denotes the fraction of the total power  $P$  allocated to the  $i^{\text{th}}$  layer. The rate of the first codebook is chosen so that the first layer is always decoded successfully even under bad channel conditions. The receiver decodes the layers sequentially using successive interference cancellation. Note that the pmf of the  $i^{\text{th}}$  layer channel capacity is a function of  $\eta_1, \dots, \eta_M$  (assuming that layers  $1, \dots, i-1$  were decoded successfully).

We now describe how SPC can be used to improve the throughput of bursty interference channels. The signal model is given by

$$\mathbf{y}_m = \sum_{i=1}^M \sqrt{\eta_i P} \mathbf{x}_{i,m} + \sum_{j=1}^N b_{j,m} \mathbf{w}_{j,m} + \mathbf{v}_m \quad (8)$$

where  $\mathbf{x}_{i,m}$  represents the  $i^{\text{th}}$  layer's block of  $L$  symbols.

**Definition 4.1 (SPC MTBRS)** *By design,  $R_1$  is chosen so that  $CW_1$ , the first codeword, is decodable under all channel conditions and treating all other codewords as noise. This means that following assignment*

$$R_1 = R_1(\boldsymbol{\eta}) = \log_2 \left( 1 + \frac{\eta_1 P}{\sum_{i=2}^M \eta_i P + \sum_j I_j + N_0} \right) \quad (9)$$

*must hold. On the other hand,  $\mathbf{R} = \{R_2, \dots, R_M\}$  and  $\boldsymbol{\eta} = \{\eta_1, \dots, \eta_M\}$  are chosen according to the following optimization problem*

$$(\mathbf{R}^*, \boldsymbol{\eta}^*) = \underset{\mathbf{R}, \boldsymbol{\eta} \text{ s.t. } \sum \eta_i = 1}{\operatorname{argmax}} T(\mathbf{R}, \boldsymbol{\eta}) \quad (10)$$

Observe that  $\mathbf{R}^*$  is necessarily a function of  $\boldsymbol{\eta}^*$  and that TBRS is a special case of SPC MTBRS for  $M = 1$  and  $\eta_1 = 1$ . This shows that SPC MTBRS will be better than TBRS.

**Theorem 4.2 (1-jammer SPC MTBRS)** *For the 1-jammer channel, the solution to the  $M = 2$  SPC MTBRS problem is given by*

$$R_1^* = \log_2 \left( 1 + \frac{\eta^* \gamma_b}{1 + (1 - \eta^*) \gamma_b} \right) \quad (11)$$

$$R_2^* = \log_2 (1 + (1 - \eta^*) \gamma_g) \quad (12)$$

$$\eta^* = 1 - \min \left( \max \left( \frac{(1 - \alpha)}{\alpha \gamma_b} - \frac{1}{\alpha \gamma_g}, 0 \right), 1 \right) \quad (13)$$

*The throughput achieved by this scheme is  $R_1^* + (1 - \alpha)R_2^*$ . Moreover, any  $M > 2$  does not provide additional throughput gains.*

**Proof 4.3** *Omitted for space limitations.*

This result says that no more than 2 layers are needed when the channel can only be in one of two conditions ( $C_g$  and  $C_b$ ). Furthermore, one can show that this scheme selects  $M = 1$  with  $R^* = C_g$  for  $\alpha \leq \frac{\gamma_g - \gamma_b}{\gamma_g \gamma_b + \gamma_g} = \alpha_1$  and  $M = 1$  with  $R^* = C_b$  for  $\alpha \geq \frac{\gamma_g - \gamma_b}{\gamma_g} = \alpha_2$ . Also, note that  $\alpha_1 \leq \alpha^* \leq \alpha_2$ . This means that there is a small range of duty cycles where SPC performs better than TBRS for the 1-jammer channel. We conjecture that more layers are needed and that the gap between the performance of SPC MTBRS and MTBRS becomes wider for the more general  $N$ -jammer channels.

##### 4.2. SPC-HARQ Methods

We now present an interference-aware scheme that combines the powers of SPC and HARQ protocols. The basic idea is to transmit  $M_1$  codewords simultaneously for a maximum of  $M_2$  times. In what follows, we specialize the discussion for  $M_1 = M_2 = 2$  as it simplifies the presentation of this hybrid technique. In this case, the transmitter uses three codebooks. A first codebook  $C_{1,1}$ , which encodes a message of  $n_{1,1}$  bits using  $L$  symbols, and hence  $R_{1,1} = n_{1,1}/L$ . A second codebook  $C_{1,2}$ , which encodes a message of  $n_{1,2}$  bits using  $L$  symbols, and hence  $R_{1,2} = n_{1,2}/L$ . A third codebook  $C_2$ , which encodes a message of  $n_2$  bits using  $2L$  symbols. We define  $R_2$  to be equal to  $n_2/L$ . The  $2L$  symbols of codewords in  $C_2$  are divided into two groups to form two "sub-codes"  $C_{2,1}$  and  $C_{2,2}$ , of length  $L$  symbols each. The receiver has knowledge of all three codebooks. The transmitter starts by transmitting a

codeword  $CW_{1,1}$  from  $\mathcal{C}_{1,1}$  simultaneously with a codeword  $CW_{2,1}$  from  $\mathcal{C}_{2,1}$ . As was the case for the regular SPC,  $\eta_1$  of  $P$  goes to  $CW_{1,1}$  and  $(1 - \eta_1)$  goes to  $CW_{2,1}$ .  $R_{1,1}$  is chosen so that  $CW_{1,1}$  is always decoded (even when seeing  $CW_{2,1}$  as interference). The receiver decodes  $CW_{1,1}$ , subtracts it out from the received block, and then decodes  $CW_{2,1}$ . A re-transmission occurs only when the decoding of  $CW_{2,1}$  fails. In this case, a new codeword  $CW_{1,2}$  is chosen from  $\mathcal{C}_{1,2}$  and transmitted simultaneously with  $CW_{2,2}$ , the second sub-codeword from  $\mathcal{C}_{2,2}$ . In the second transmission, the power assigned to  $CW_{1,2}$  is  $\eta_2$ , which could possibly be different from  $\eta_1$ , and the power assigned to  $CW_{2,1}$  is  $1 - \eta_2$ . The fact that the power assignment is not fixed across transmissions will turn out to be beneficial as we will see in the results section.  $R_{1,2}$  is also chosen so that  $CW_{1,2}$  is always decoded successfully. For the N-jammer interference channel, the following hybrid SPC HARQ signal model holds

$$\mathbf{y}_m^i = \sqrt{\eta_i P} \mathbf{x}_{1,m}^i + \sqrt{(1 - \eta_i) P} \mathbf{x}_{2,m}^i + \sum_{j=1}^N b_{j,m} \mathbf{w}_{j,m} + \mathbf{v}_m \quad (14)$$

where  $i \in \{1, 2\}$  represents the transmission number. By definition,  $R_{1,i}$  is chosen so that the first codeword is decodable under all channel conditions. This means that the following rate assignment

$$R_{1,i} = R(\eta_i) = \log_2 \left( 1 + \frac{\eta_i P}{(1 - \eta_i) P + \sum_j I_j + N} \right) \quad (15)$$

must hold for the proper operation of this scheme. We define the random variable  $C_i(\eta_i)$ ,  $i \in \{1, 2\}$ , to represent the channel capacity after decoding and subtracting out  $CW_{1,i}$ . Using the renewal-reward theorem, the throughput of this hybrid SPC-HARQ is given by

$$T(R_2, \eta_1, \eta_2) = \frac{R(\eta_1) + P(R_2 \geq C_1(\eta_1)) R(\eta_2)}{\bar{m}(R_2, \eta_1)} \quad (16)$$

$$+ \frac{R_2(1 - \delta(R_2, \eta_1, \eta_2))}{\bar{m}(R_2, \eta_1)}$$

$$\delta(R_2, \eta_1, \eta_2) = P(R_2 \geq C_1(\eta_1) + C_2(\eta_2))$$

$$\bar{m}(R_2, \eta_1) = 2 - P(R_2 < C_1(\eta_1))$$

where  $\delta(R_2, \eta_1, \eta_2)$  represents the outage probability for the  $2^{nd}$  layer and  $\bar{m}(R_2, \eta_1)$  represents the average number of re-transmissions.

**Definition 4.4 (SPC-HARQ MTBRS)**  $R_2$ ,  $\eta_1$ , and  $\eta_2$  are chosen according to the following optimization problem

$$(R_2^*, \eta_1^*, \eta_2^*) = \underset{R_2, \eta_1, \eta_2 \text{ s.t. } \eta_1, \eta_2 \leq 1}{\operatorname{argmax}} T(R_2, \eta_1, \eta_2) \quad (17)$$

Unfortunately, we could not find a closed form expression for  $(R_2^*, \eta_1^*, \eta_2^*)$ . However, an optimal (possibly non-unique) solution always exists because all the variables we are optimizing over, including  $R_2$ , are bounded. This means that the

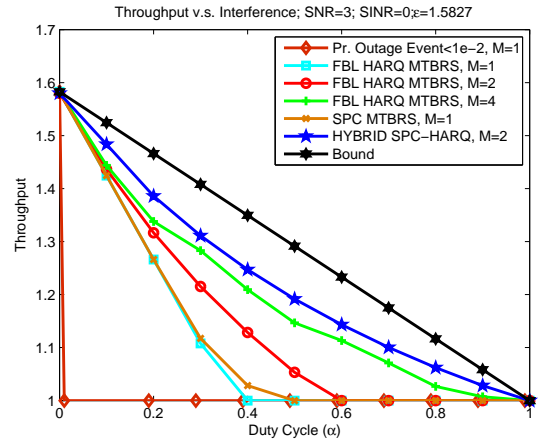


Fig. 1. SNR=3 dB, SINR=0 dB: Throughput v.s.  $\alpha$

SPC-HARQ MTBRS problem can be solved using a greedy search algorithm.

## 5. RESULTS

Figure 1 summarizes the performance of all schemes for a 1-jammer channel with  $C_g/C_b \approx 1.5$ . The bound represents the average capacity of this channel for various duty cycles  $\alpha$ . Several observations are in order. First, a traditional system that is designed such that the probability of an outage event is kept below  $\delta = 0.01$  has poor performance in the presence of bursty interference. Second, as discussed in the previous section, the improvement in performance due to SPC is limited to a very small range of duty cycles ( $\alpha \in [\alpha_1, \alpha_2]$ ). Outside this range, the performance of TBRS and SPC MTBRS are identical. Third, the performance of HARQ MTBRS improves with  $M$  and for any  $M > 1$ , HARQ MTBRS is better than SPC MTBRS. However, this comes at the expense of decoding complexity because in the worst case, a code of length  $ML$  symbols has to be decoded. Finally, the hybrid SPC-HARQ scheme, which re-transmits once (if needed), outperforms HARQ MTBRS with  $M = 4$  for all duty cycles.

## 6. CONCLUSION

This paper proposed and analyzed novel rate selection algorithms for bursty interference channels. SPC-HARQ based rate control schemes were shown to outperform optimized single-layer HARQ schemes. It was also shown through simulations, that the proposed SPC-HARQ scheme approaches the bound on the channel capacity. Future work includes studying this problem with fading channels in addition to designing variable block length HARQ schemes that can re-transmit  $\beta L$  symbols (with  $\beta < 1$ ) instead of  $L$  symbols.

## 7. REFERENCES

- [1] A. Gudipati, S. Pereira, and S. Katti, "Automac: rateless wireless concurrent medium access," in *Proceedings of the 18th annual international conference on Mobile computing and networking*. ACM, 2012, pp. 5–16.
- [2] A. Gudipati and S. Katti, "Strider: Automatic rate adaptation and collision handling," *SIGCOMM-Computer Communication Review*, vol. 41, no. 4, p. 158, 2011.
- [3] P. Frenger, S. Parkvall, and E. Dahlman, "Performance comparison of HARQ with chase combining and incremental redundancy for HSDPA," in *Vehicular Technology Conference, 2001. VTC 2001 Fall. IEEE VTS 54th*, vol. 3. IEEE, 2001, pp. 1829–1833.
- [4] G. Caire and D. Tuninetti, "The throughput of hybrid-ARQ protocols for the gaussian collision channel," *Information Theory, IEEE Transactions on*, vol. 47, no. 5, pp. 1971–1988, Jul, 2001.
- [5] R. Wolff, *Stochastic modeling and the theory of queues*. Prentice hall Englewood Cliffs, NJ, 1989, vol. 14.
- [6] T. Cover, "Broadcast channels," *Information Theory, IEEE Transactions on*, vol. 18, no. 1, pp. 2–14, 1972.
- [7] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley-Interscience, 2006.