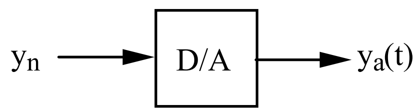
ECE 410 **DIGITAL SIGNAL PROCESSING** **D. Munson**
University of Illinois **Chapter 9**

Implementation of Ideal D/A

Consider



Recall that any D/A we encounter in this course can be modeled by

$$y_a(t) = \sum_{n=-\infty}^{\infty} y_n \, g_a(t - nT) \tag{1}$$

and that the Fourier-domain relation is

$$Y_a(\Omega) = G_a(\Omega) \, Y_d(\Omega T) \tag{2}$$

For the ideal D/A, we have $g_a(t) = \text{sinc}\left(\frac{\pi}{T} t\right)$, giving

$$y_a(t) = \sum_{n=-\infty}^{\infty} y_n \;\text{sinc}\left[\frac{\pi}{T}(t - nT)\right] \tag{3}$$

and

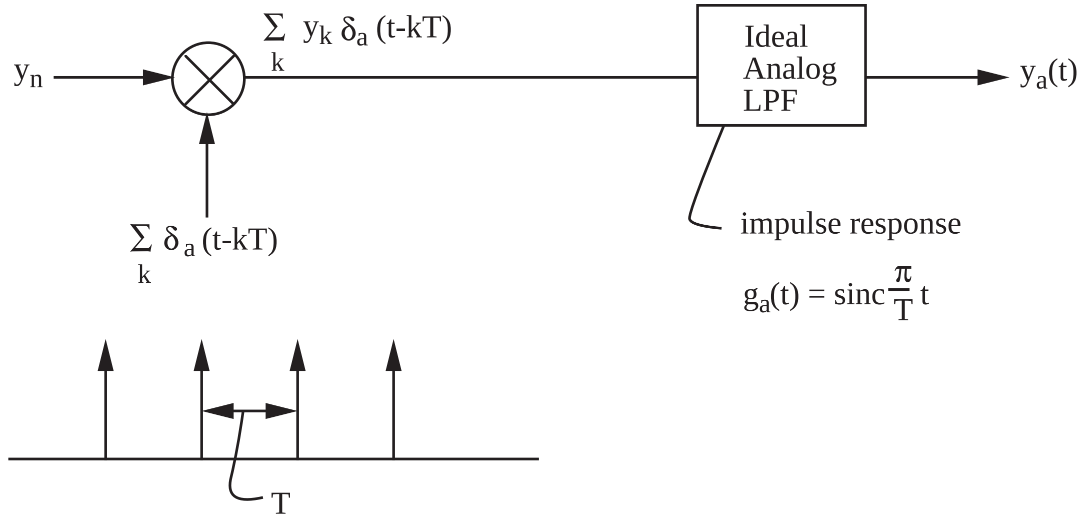
$$G_a(\Omega) = \begin{cases} T & |\Omega| \le \frac{\pi}{T} \\ 0 & \text{else} \end{cases}$$

so that (2) gives

$$Y_a(\Omega) = \begin{cases} T \, Y_d(\Omega T) & |\Omega| \le \frac{\pi}{T} \\ 0 & \text{else} \end{cases} \tag{4}$$

How might we implement the ideal D/A, described by (3)?

Conceptually, we might think along the lines of:

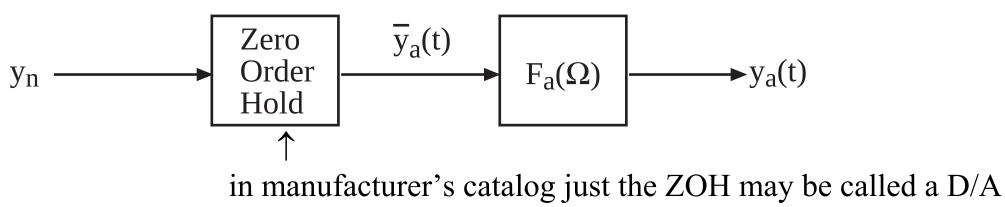


Then:

$$y_a(t) = g_a(t) * \sum_n y_n \ \delta_a(t-nT) = \sum_n y_n \ g_a(t-nT) \ \text{ as desired.}$$

For an actual implementation, we might consider approximating the impulse train by a periodic sequence of very tall, narrow pulses. **However, this would be difficult in practice. As a result, D/A's are not implemented as suggested above!**

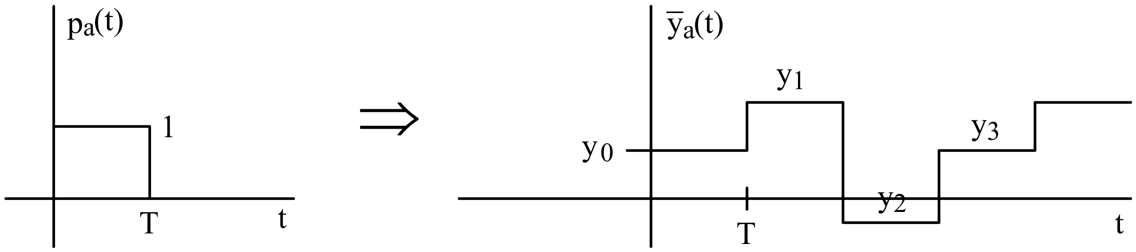
In practice, the ideal D/A is approximated with the following two-stage system:



What is a zero-order hold (ZOH)? It is a D/A that uses rectangular pulses, i.e.,

$$\bar{y}_a(t) = \sum_n y_n \, p_a(t - nT) \tag{5}$$

where



Thus, the ZOH output is a staircase approximation to the desired $y_a(t)$. This staircase must be smoothed by $F_a(\Omega)$ to produce the proper $y_a(t)$.

The Fourier-domain relation for the ZOH has the form given by (2), but now

$$
\begin{aligned}
G_a(\Omega) &= \int_0^T 1 \bullet e^{-j\Omega t}\, dt \\
&= \left. \frac{e^{-j\Omega t}}{-j\Omega} \right|_0^T = \frac{e^{-j\Omega T} - 1}{-j\Omega} \\
&= \frac{e^{-j\Omega \frac{T}{2}} \left(e^{-j\Omega \frac{T}{2}} - e^{j\Omega \frac{T}{2}} \right)}{-j\Omega} = e^{-j\Omega \frac{T}{2}} \, \frac{2 \sin \frac{\Omega T}{2}}{\Omega} \\
&= T\, e^{-j\Omega \frac{T}{2}} \operatorname{sinc} \frac{\Omega T}{2}
\end{aligned}
$$

Thus,

$$
\overline{Y}_a(\Omega) = T\, e^{-j\Omega \frac{T}{2}} \operatorname{sinc} \left(\frac{\Omega T}{2} \right) Y_d(\Omega T) \tag{6}
$$

Before deciding how to choose $F_a(\Omega)$, which follows the ZOH, let's see how the effect of the ZOH differs from the ideal D/A, in the Fourier domain.

Example

Suppose have

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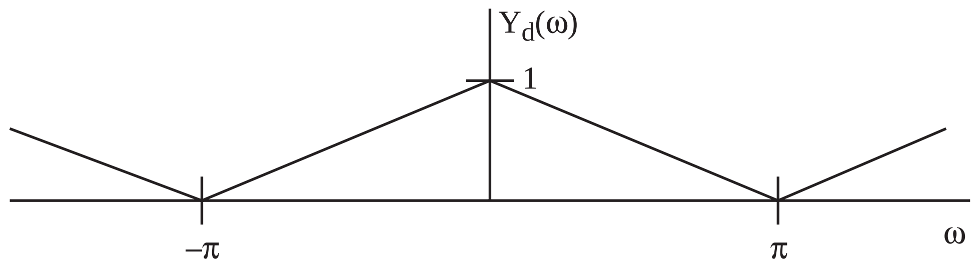
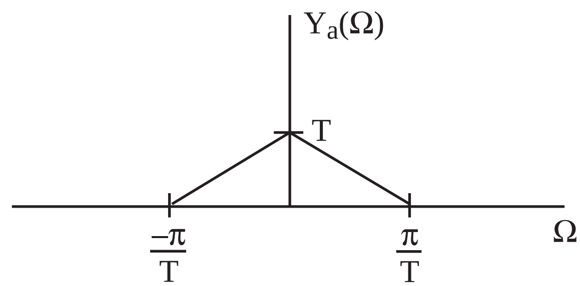
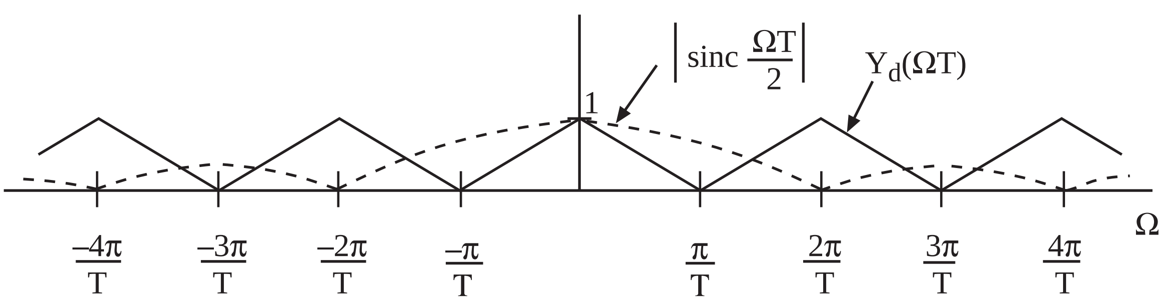


Sketch $Y_a(\Omega)$, the Fourier transform of the output of an ideal D/A, and $\overline{Y}_a(\Omega)$, the Fourier transform of the output of a ZOH.

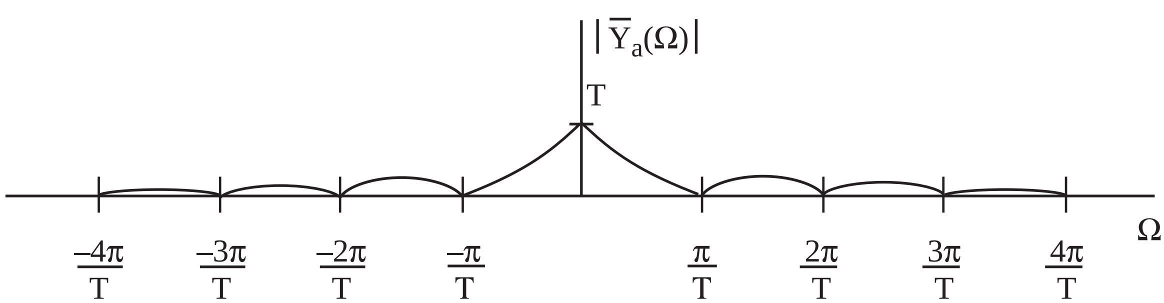
Using (4), we have for the ideal D/A:



For the ZOH, let's plot $|\overline{Y}_a(\Omega)|$. The terms in (6) look like:



$|\overline{Y}_a(\Omega)|$ is T times the product of the above two curves:



Notice that unlike $Y_a(\Omega)$ for the ideal D/A, $|\overline{Y}_a(\Omega)|$ for the ZOH has frequency content that extends all the way to $\Omega = \pm\infty$. This is not surprising, since $\overline{y}_a(t)$, for the ZOH, is a staircase function with discontinuities. Sharp edges (discontinuities) always correspond to a frequency content extending to $\pm\infty$.

Now, if we have $\overline{Y}_a(\Omega)$ from the ZOH, how do we choose $F_a(\Omega)$ to produce $Y_a(\Omega)$? The above sketches suggest that we need $F_a(\Omega)$ to be a LPF with cutoff at $\Omega_c = \pm\frac{\pi}{T}$. To investigate this thoroughly, note that for the ZOH system we have

$$\begin{aligned} Y_a(\Omega) &= F_a(\Omega)\,\overline{Y}_a(\Omega) \\ &= F_a(\Omega)\,T\,e^{-j\frac{\Omega T}{2}}\,\text{sinc}\left(\frac{\Omega T}{2}\right)\,Y_d(\Omega T) \end{aligned} \qquad (7)$$

For the ideal D/A, the relation is given by (4). To have (7) correspond to (4) we must have

$$F_a(\Omega)\,T\,e^{-j\frac{\Omega T}{2}}\,\text{sinc}\left(\frac{\Omega T}{2}\right)\,Y_d(\Omega T) = \begin{cases} T\,Y_d(\Omega T) & |\Omega| \le \frac{\pi}{T} \\ 0 & \text{else} \end{cases}$$

or

$$\boxed{F_a(\Omega) = \begin{cases} \dfrac{e^{j\frac{\Omega T}{2}}}{\text{sinc}\left(\frac{\Omega T}{2}\right)} & |\Omega| \le \frac{\pi}{T} \\ 0 & |\Omega| > \frac{\pi}{T} \end{cases}}$$

The first zero-crossing of sinc $\frac{\Omega T}{2}$ occurs when $\frac{\Omega T}{2} = \pi \Rightarrow \Omega = \frac{2\pi}{T}$.

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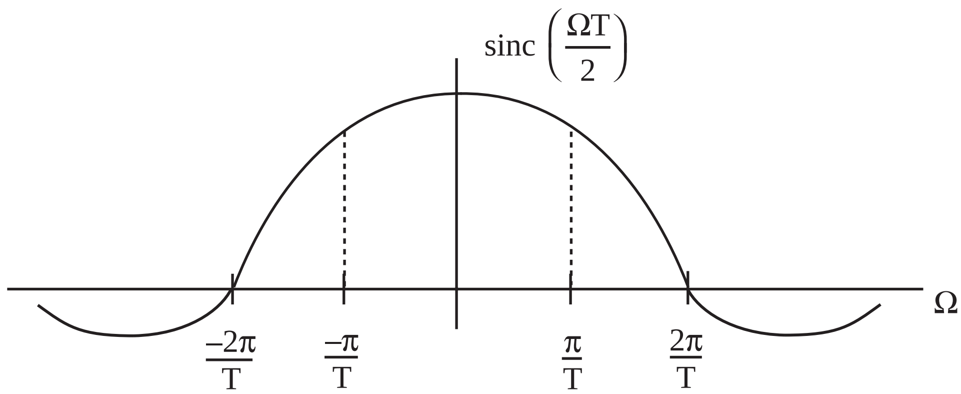
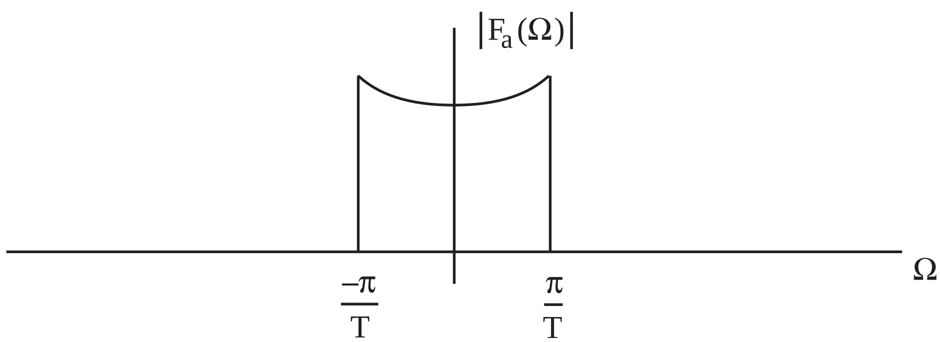
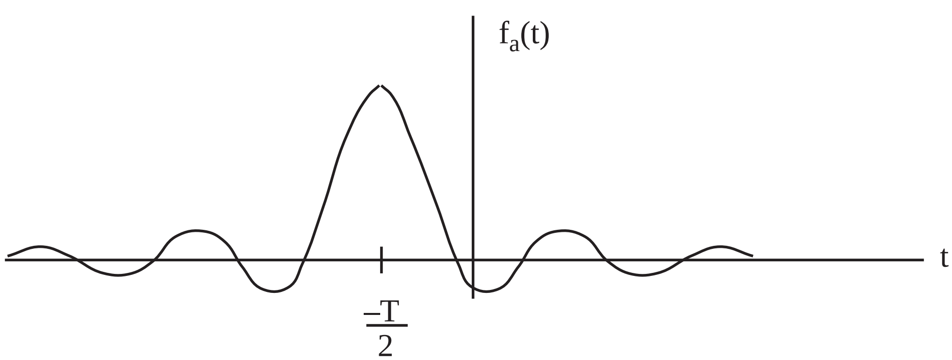


So, $|F_a(\Omega)|$ looks like:

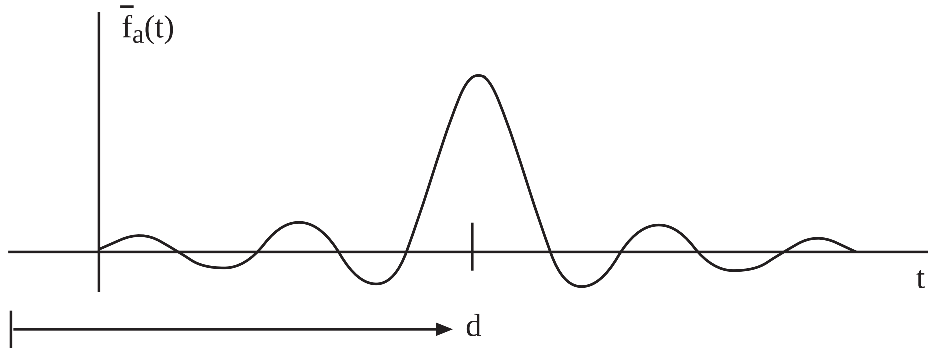


Thus, the ideal $F_a(\Omega)$ is a LPF that emphasizes the higher frequencies in its passband. (Surprising!)

$F_a(\Omega)$ has finite support $\Rightarrow$ $f_a(t)$ has infinite support. $f_a(t)$ might look something like:



In practice, we would use a filter with a causal impulse response $\bar{f}_a(t)$ with $\bar{f}_a(t) \approx f_a(t-d)\, u_a(t)$ (delayed and truncated version of $f_a(t)$).

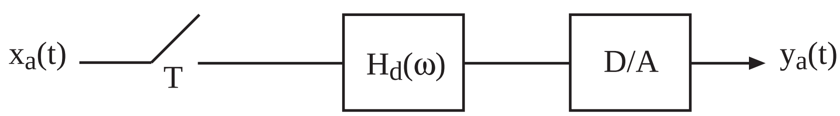


Using $\bar{f}_a(t)$ will delay the desired output by d seconds, but this is no problem in most applications if d is small.

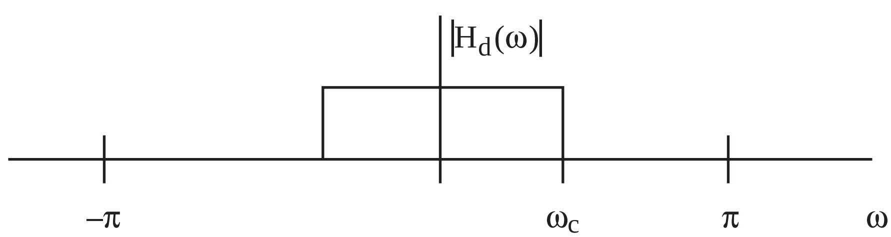
<u>Notes</u>:

1. In cheaper D/As, we may use a very simple R-C network to crudely approximate the desired $F_a(\Omega)$.

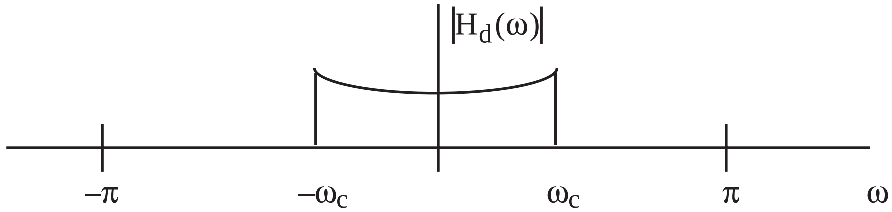
2. The high-frequency emphasis within the passband of $F_a(\Omega)$ can be performed <u>digitally</u> as part of the digital filter function. For example, if wish to realize an analog LPF using



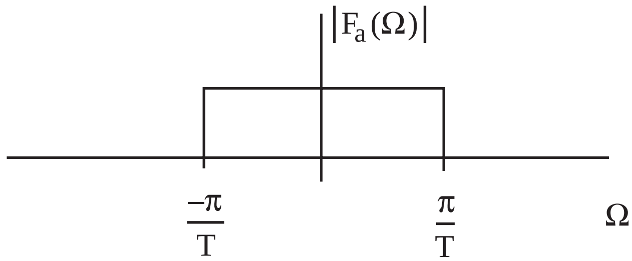
then instead of using



could use



In this case, we still need an $F_a(\Omega)$ after the ZOH, but now $F_a(\Omega)$ can be a regular LPF with a flat response in the passband:

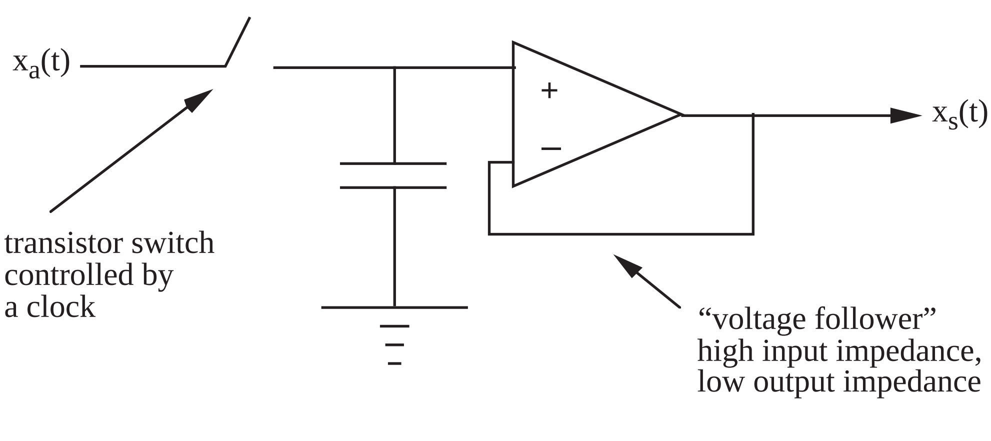


A/D and D/A Circuits

A/D consists of sample and hold followed by a quantizer.

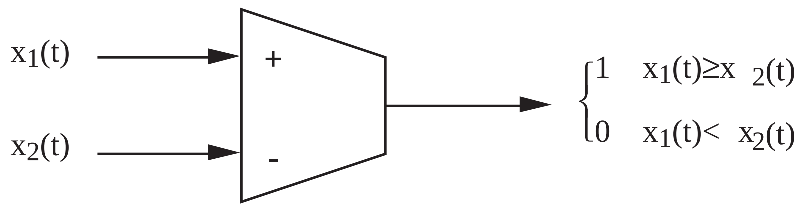
In catalogs, just the quantizer is called an A/D (unless A/D is referred to as a "sampling A/D"). As we shall see, the sample and hold is very simple, whereas the quantizer is much more complicated.

Sample and Hold:



A/D (Quantizer)

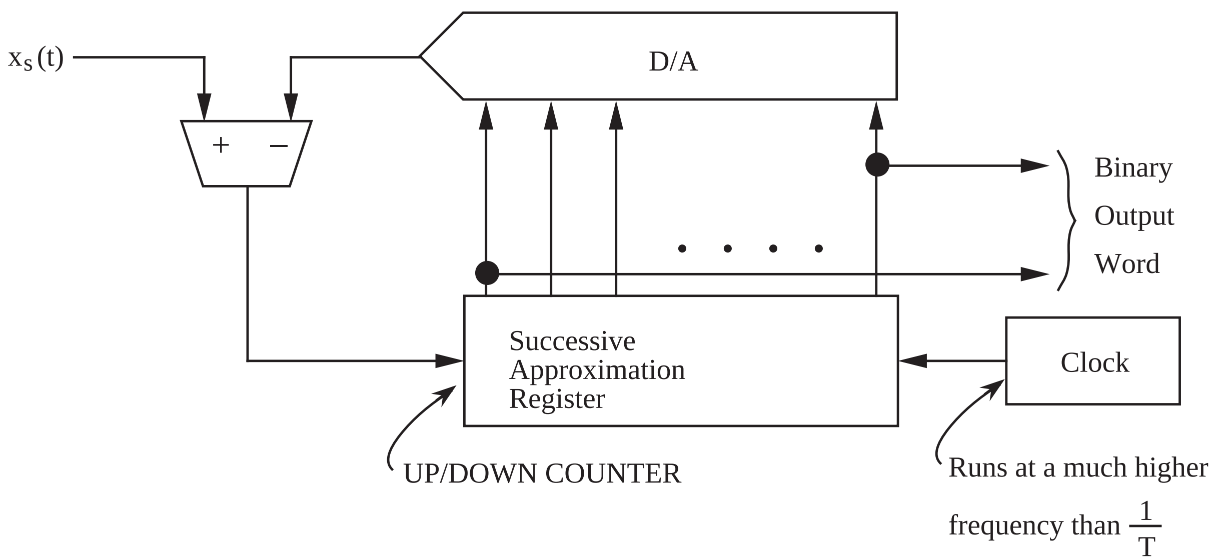
Uses comparators:



Two popular types of A/D's:

a) Successive Approximation

~ for low and medium sampling rates; uses a D/A!



Here, $x_s(t)$ is the input from the sample and hold. The above system quantizes $x_s(t)$ to fit into a computer register. The comparator output signal causes the up-down counter to either increment or decrement, at a high rate, until it contains a binary approximation of $x_s(t)$. When the counter has settled around the correct digital representation of $x_s(t)$, it simply toggles back and forth in its least significant bit until the value of $x_s(t)$ changes.

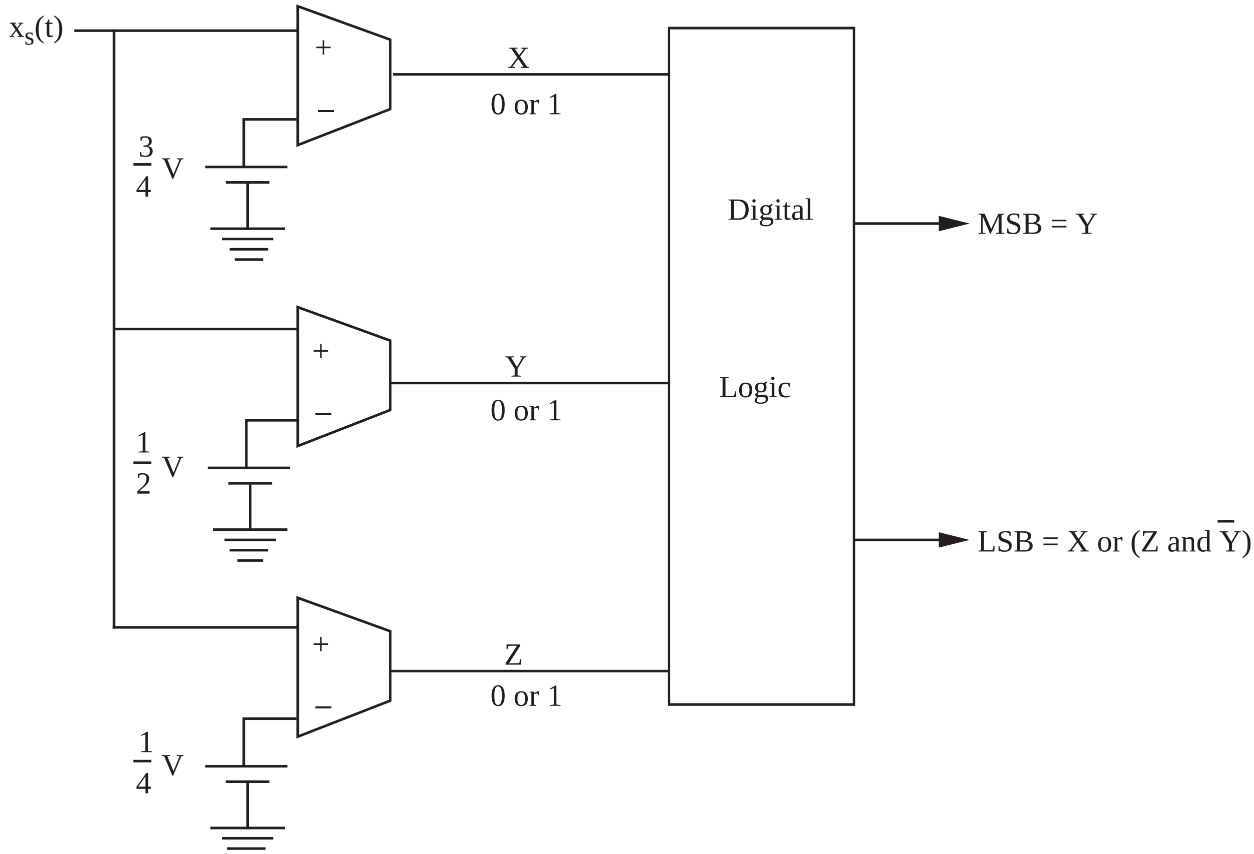
Succcessive approximation A/D's are fairly slow (and thus used for low and medium bandwidth applications) because it may take several clock cycles for the counter to settle on a new value of $x_s(t)$.

b) Parallel or Flash A/D

For high speed (8 bits/sample at 500 MHz is currently possible).

Uses 2^N-1 comparators for N-bit output word.

Example 2 bit quantizer:

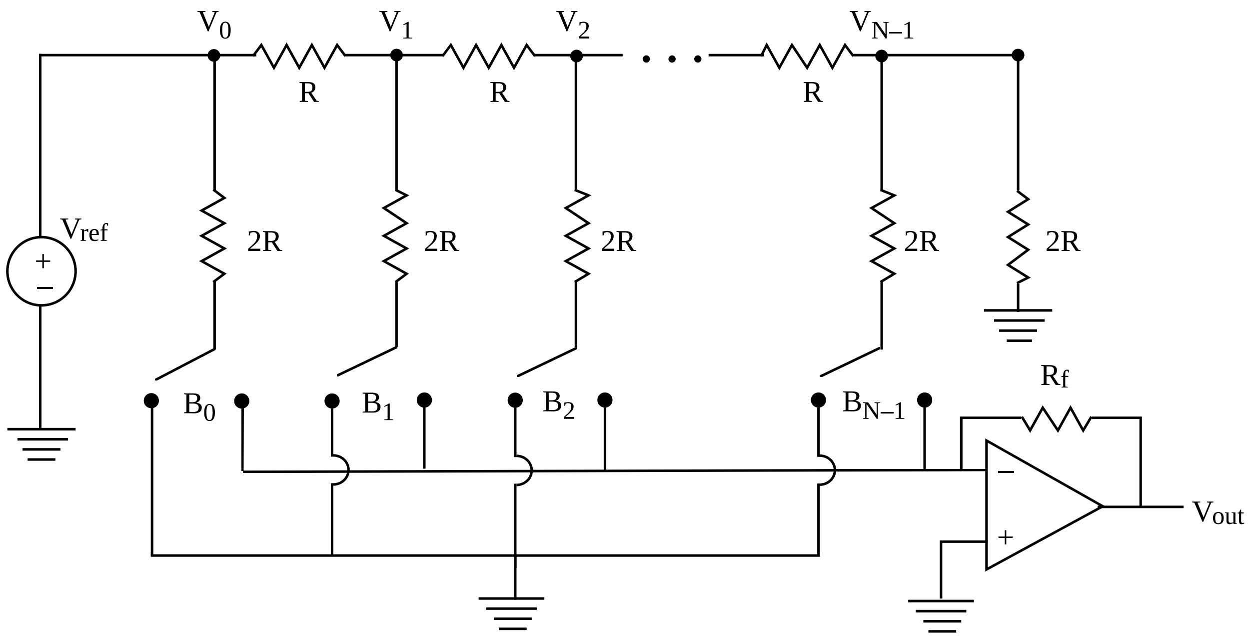


Here, $0 \le x_s(t) < \frac{1}{4}$ is mapped to (0, 0), $\frac{1}{4} \le x_s(t) < \frac{1}{2}$ is mapped to (0, 1), $\frac{1}{2} \le x_s(t) < \frac{3}{4}$ is mapped to (1, 0), and $x_s(t) \ge \frac{3}{4}$ is mapped to (1, 1).

D/A Converters ~ Zero Order Hold (ZOH)

The contents of a binary register containing y_n are the input to a D/A. Let $\left[B_0, B_1, B_2, \ldots, B_{N-1}\right]$ be the binary representation of y_n. The B_i change with period T as y_{n+1} replaces y_n in the D/A input register.

One popular type of D/A uses a resistor ladder (can also use a capacitor ladder):



The switches are transistors, where the B_i control whether the transistors conduct to ground (left position, $B_i = 0$) or to the op amp (right position, $B_i = 1$). The op amp then adds all signals input to its minus terminal, with a weighting determined by the resistor values. To find the exact relationship between V_{out} and the B_i, first apply KCL at Node N–1 at the upper right, to give:

$$\frac{V_{N-1}}{2R} + \frac{V_{N-1}}{2R} + \frac{V_{N-1} - V_{N-2}}{R} = 0$$

$$\Rightarrow \; V_{N-1} + V_{N-1} - V_{N-2} = 0 \Rightarrow \; V_{N-2} = 2\, V_{N-1}$$

Similarly: $V_{n-1} = 2V_n \quad n = 1, 2, \; \ldots, N-2$

$$\Rightarrow V_n = V_{N-1}\, 2^{N-1-n}$$

Using KCL at the minus terminal of the op amp gives:

$$\frac{1}{2R} \sum_{i=0}^{N-1} B_i\, V_i = \frac{0 - V_{out}}{R_f}$$

where each B_i is 0 or 1.

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$$\Rightarrow \qquad V_{out} = \frac{-R_f}{2R} \sum_{i=0}^{N-1} B_i V_{N-1} 2^{N-1-i}$$

$$= \frac{-R_f}{2R} V_{N-1} \left[2^{N-1} B_0 + 2^{N-2} B_1 + \ldots + 2 B_{N-2} + B_{N-1} \right]$$

So, V_{out} is proportional to the number stored in the binary register representing y_n. This number changes according to a clock ($y_n \rightarrow y_{n+1}$), so $V_{out}(t)$ is a staircase function (edges won't be perfectly square, though — op amp has a nonzero rise time).

The ZOH is followed with the analog LPF, below, as discussed previously.