UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN Department of Electrical and Computer Engineering ECE 410 DIGITAL SIGNAL PROCESSING

Quiz Number 1

Thursday, September 6, 2007

Student Name:

Section: Prof. Bresler / Prof. Singer

NOTE: You may not use any calculators or notes on this quiz

Problem 1 (10 points) Given that

$$u[n] = \begin{cases} 1 & n \ge 0\\ 0 & n < 0 \end{cases}$$

plot the following discrete-time signal. Label the horizontal and vertical axes for full credit.

$$x[n] = nu[-n+6]u[n-2]$$

Problem 2

(a) (15 points)

Determine the discrete-time Fourier Transform (DTFT) $X_d(\omega)$ of the following sequence.

$$x[n] = \begin{cases} (-1)^n & 0 \le n \le 4\\ 0 & \text{otherwise} \end{cases}$$

Express $X_d(\omega)$ as $X_d(\omega) = R(\omega)e^{j\phi(\omega)}$, where $R(\omega)$ is a purely real function of ω .

(b) (15 points) Evaluate $X_d(\omega)$ at $\omega = \frac{\pi}{2}$, $\omega = \frac{2\pi}{3}$, $\omega = \pi$. Express your solutions in magnitude/phase form (i.e., $Ae^{j\phi}$, where A is a positive real number).

Problem 3 (20 points) Given that the DTFT for a *real* discrete-time signal x[n] is:

$$X_{d}[\omega] = \begin{cases} 6[\cos(-\frac{2\pi}{3}) + j\sin(\frac{2\pi}{3})] & \text{for } \omega = -\frac{\pi}{7} \\ 12[\cos(\frac{\pi}{4}) - j\sin(-\frac{\pi}{4})] & \text{for } \omega = \frac{3\pi}{7} \\ ???? & \text{elsewhere on } [-\pi, \pi] \end{cases}$$

find $X_d(\frac{\pi}{7})$ and $X_d(-\frac{3\pi}{7})$. Give your solution(s) both in rectangular and polar form.

Problem 4 (20 points) Given that the DTFT of a certain signal x[n] is:

$$X_d[\omega] = \begin{cases} 2 & \text{for } |\omega| \le \frac{\pi}{2} \\ 0 & \text{elsewhere on } [-\pi, \pi] \end{cases}$$

find the DTFT of $y[n] = x[n]\cos(\frac{\pi}{2}n)$ on $[-\pi,\pi]$. Determine a simplified expression for y[n].

Problem 5 (20 points) Simplify the following expression

$$\frac{(1-j)^8}{8(\sqrt{3}-j)} + \frac{e^{j\frac{\pi}{6}}}{e^{j\pi}} - (1+j)$$

Give your solution both in rectangular and polar form.