UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering ECE 410 Digital Signal Processing

## Quiz Number 1

Thursday, September 6, 2007
Student Name:
Section: Prof. Bresler / Prof. Singer

NOTE: You may not use any calculators or notes on this quiz

Problem 1 (10 points) Given that

$$
u[n]= \begin{cases}1 & n \geq 0 \\ 0 & n<0\end{cases}
$$

plot the following discrete-time signal. Label the horizontal and vertical axes for full credit.

$$
x[n]=n u[-n+6] u[n-2]
$$

## Problem 2

(a) (15 points)

Determine the discrete-time Fourier Transform (DTFT) $X_{d}(\omega)$ of the following sequence.

$$
x[n]= \begin{cases}(-1)^{n} & 0 \leq n \leq 4 \\ 0 & \text { otherwise }\end{cases}
$$

Express $X_{d}(\omega)$ as $X_{d}(\omega)=R(\omega) e^{j \phi(\omega)}$, where $R(\omega)$ is a purely real function of $\omega$.
(b) (15 points) Evaluate $X_{d}(\omega)$ at $\omega=\frac{\pi}{2}, \omega=\frac{2 \pi}{3}, \omega=\pi$. Express your solutions in magnitude/phase form (i.e., $A e^{j \phi}$, where $A$ is a positive real number).

Problem 3 (20 points) Given that the DTFT for a real discrete-time signal $x[n]$ is:

$$
X_{d}[\omega]= \begin{cases}6\left[\cos \left(-\frac{2 \pi}{3}\right)+j \sin \left(\frac{2 \pi}{3}\right)\right] & \text { for } \omega=-\frac{\pi}{7} \\ 12\left[\cos \left(\frac{\pi}{4}\right)-j \sin \left(-\frac{\pi}{4}\right)\right] & \text { for } \omega=\frac{3 \pi}{7} \\ ? ? ? ? & \text { elsewhere on }[-\pi, \pi]\end{cases}
$$

find $X_{d}\left(\frac{\pi}{7}\right)$ and $X_{d}\left(-\frac{3 \pi}{7}\right)$. Give your solution(s) both in rectangular and polar form.

Problem 4 (20 points) Given that the DTFT of a certain signal $x[n]$ is:

$$
X_{d}[\omega]= \begin{cases}2 & \text { for }|\omega| \leq \frac{\pi}{2} \\ 0 & \text { elsewhere on }[-\pi, \pi]\end{cases}
$$

find the DTFT of $y[n]=x[n] \cos \left(\frac{\pi}{2} n\right)$ on $[-\pi, \pi]$. Determine a simplified expression for $y[n]$.

Problem 5 (20 points) Simplify the following expression

$$
\frac{(1-j)^{8}}{8(\sqrt{3}-j)}+\frac{e^{j \frac{\pi}{6}}}{e^{j \pi}}-(1+j)
$$

Give your solution both in rectangular and polar form.

