University of Illinois **Spring 2009**

ECE 410 **Profs. Ahuja & Liang**

Midterm Exam I

Thursday, February 26, 2009

Problem	Pts.	Score
1	10	
2	16	
3	6	
4	12	
5	6	
6	13	
7	4	
8	18	
9	15	
Total	100	

Name __________________________________

Section: 9:00 AM 2:00 PM

Score ____________

Please do not turn this page over until told to do so.

You may not use any books, calculators, or notes other than two sides of a 8.5" x 11" sheet of paper.

GOOD LUCK!

Problem 1{10 Points}
Determine the continuous-time Fourier transform (CTFT) of $x_1(t) = e^{-5|t|}$. Let it be denoted by $X_1(\omega)$. The magnitude of $X_1(\omega)$ is given in Figure 1.

1. Sketch the phase of $X_1(\omega)$ in Figure 2.
2. Let $X_2(\omega)$ denote the CTFT of $x_2(t) = e^{-5|t-3|}$. Sketch the magnitude and phase of the $X_2(\omega)$ in Figures 3 and 4.

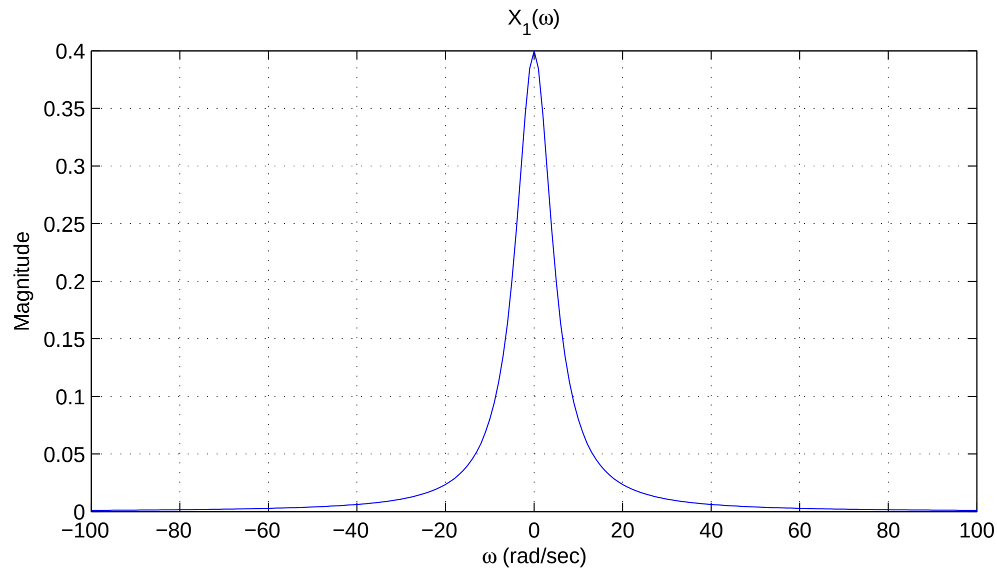


Figure 1: Magnitude of $X_1(\omega)$

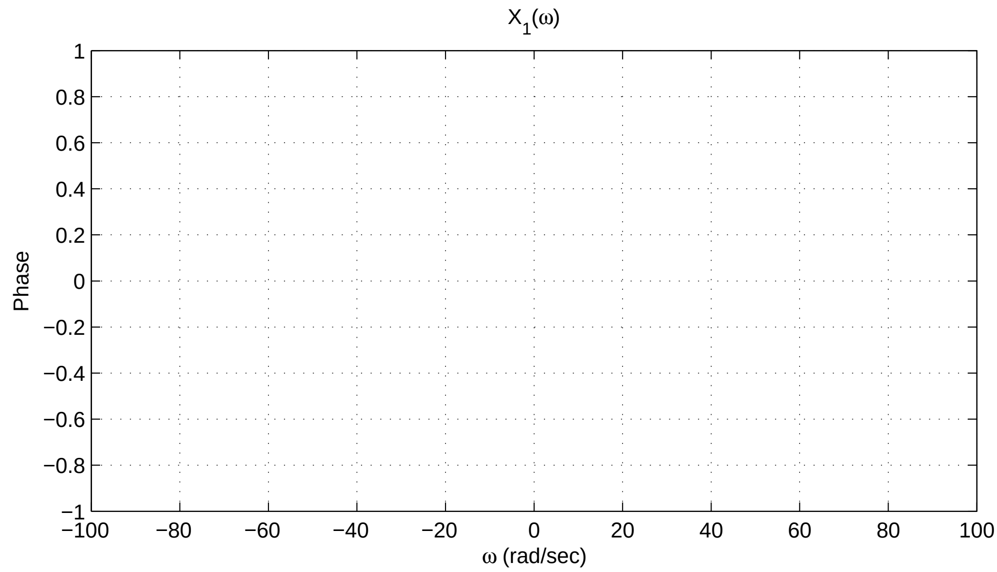


Figure 2: Phase of $X_1(\omega)$

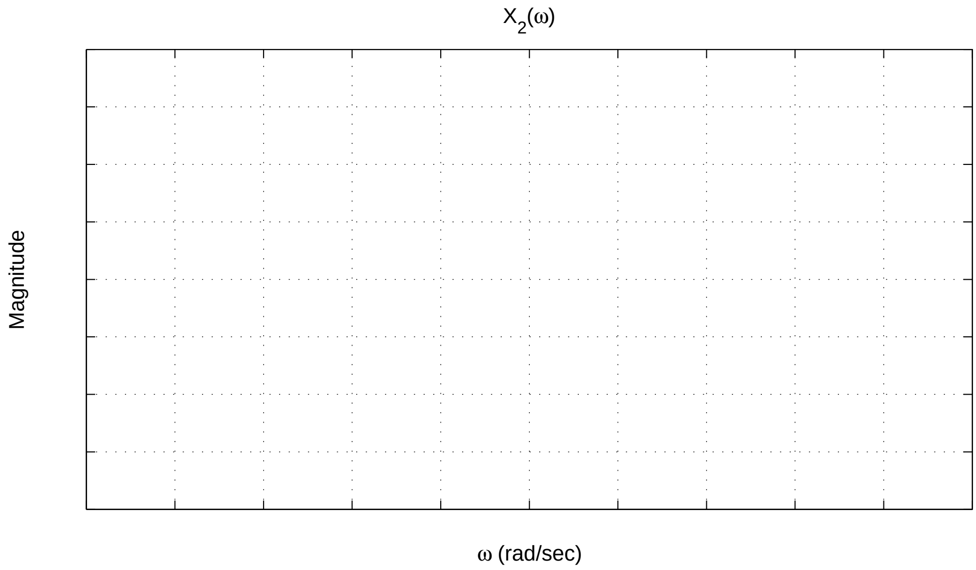


Figure 3: Magnitude of $X_2(\omega)$

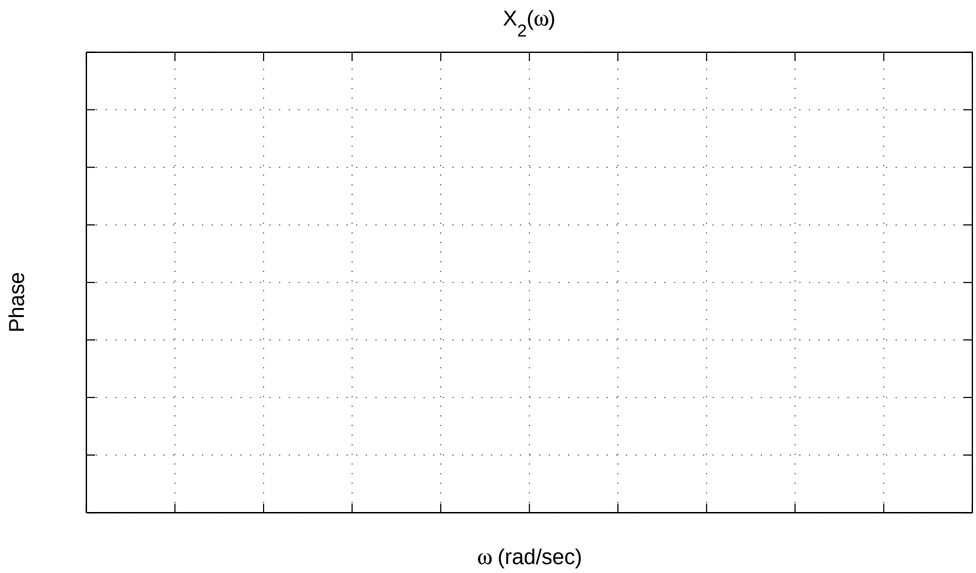


Figure 4: Phase of $X_2(\omega)$

Problem 2{16 Points}

The nonzero elements of a discrete-time sequence $x(n)$ are: $x[-3] = -1$, $x[-1] = 1$, $x[1] = 1$, $x[3] = 1$, $x[4] = 2$, $x[5] = 1$, $x[7] = -1$. For all other n, $x[n] = 0$. Calculate the following WITHOUT obtaining $X_d(\omega)$ first.

1. (a) $X_d(0)$

   (b) $\int_{-\pi}^{\pi} X_d(\omega) d\omega$

   (c) $X_d(\pi)$

   (d) $\int_{-\pi}^{\pi} |X_d(\omega)|^2 d\omega$

   (e) $\int_{-\pi}^{\pi} |\frac{dX_d(\omega)}{d\omega}|^2 d\omega$

2. Let $Y_d(\omega) = Re(X_d(\omega))$. Find the discrete-time sequence $y[n]$ whose DTFT is $Y_d(\omega)$.

Problem 3{6 Points}

The continuous-time signal $x_a(t)$ has the continuous-time Fourier transform shown in the figure below. The signal $x_a(t)$ is sampled with sampling interval T to get the discrete-time signal $x[n] = x_a(nT)$.

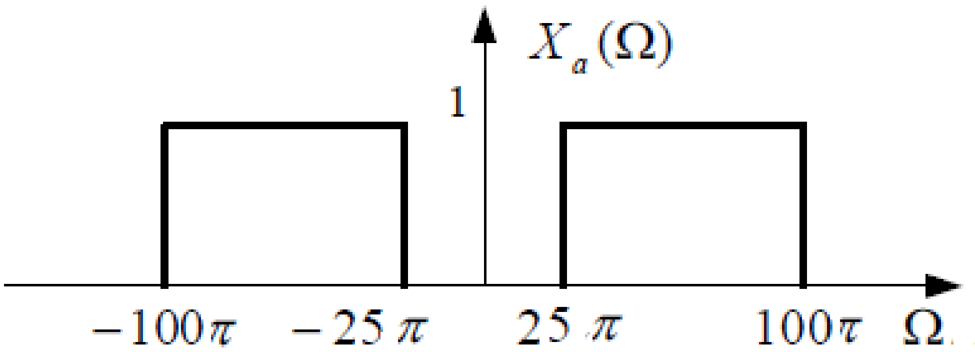
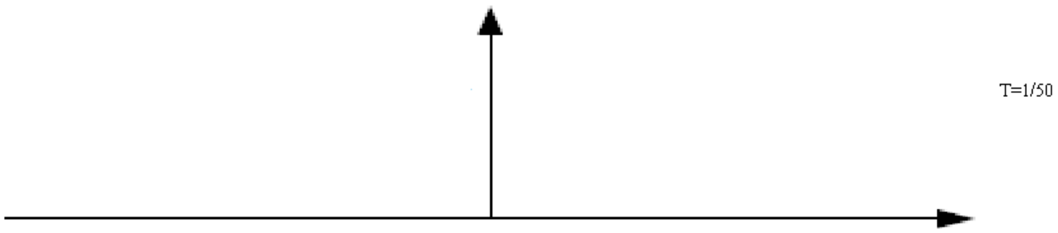
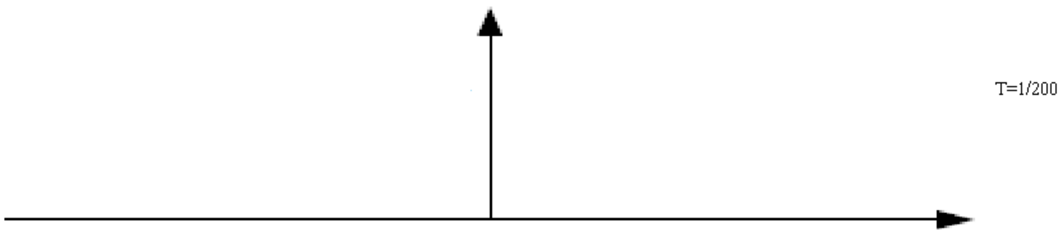


Figure 5: $X_a(\Omega)$

a. Sketch $X_d(\omega)$ (the DTFT of $x[n]$) for the sampling intervals T= 1/200 and 1/50 in the corresponding frames provided. Remember to label the axes and show associated values on the axes.





b. What is the minimum sampling rate f_s (Nyquist rate) such that no aliasing will occur in sampling the continuous-time signal?

Nyquist Rate = ______________________

Problem 4{12 Points}

Compute the discrete-time Fourier transform (DTFT) of the following signals directly using the defining formula.

(a) $x[n] = \sin(\frac{3}{4}n)$

(b) $x[n] = -u[n+3] + u[n-3]$

(c) $x[n] = (0.4e^{j\pi/2})^n u[n]$

Problem 5{6 Points}

Let $x_a(t) = \sin(7\pi t) + 0.75\cos(5\pi t)$. Let $\{X_m\}_{m=0}^{(M-1)}$ denote the order-M DFT of $x_a(t)$.

1. Given that the analog frequency corresponding to $X[51]$ is 3.984π, determine the relationship between M and T where T is the sampling period.

2. Given a 2 second long segment of $x_a(t)$, how would you choose the sampling interval T to resolve the sinusoidal components and avoid aliasing? State your criterion for resolvability.

Problem 6{13 Points}

Let $x[n]$ denote the input and $h[n]$ the impulse response of a linear time-invariant system. For the pairs of x and h given in parts (a)-(c), determine the output $y[n]$. You do not need to solve parts (b) and (c) independently; use your knowledge of linearity and time invariance to minimize the work in parts (b) and (c).

a. $x[n] = u[n]$ and $h[n] = a^n u[-n-1]$, with $|a| > 1$.

b. $x[n] = u[n-4]$ and $h[n] = 2^n u[-n-1]$.

c. $x[n] = u[n]$ and $h[n] = (0.5)2^n u[-n]$.

Problem 7{4 Points}

Give two examples where zero-padding is useful in digital signal processing.

Problem 8{18 Points}

Suppose you are given the 4-point discrete-time sequence $x[n] = \{\underset{\uparrow}{2}, 1, 2, 1\}$ where the first element corresponds to $n = 0$.

a. Compute the DFT of $X[m]$ of $x[n]$.

b. Suppose $x[n]$ is the sampled version of the continuous-time signal $x(t)$. Using standard notation, give the equation that relates $x(t)$ and $X[m]$.

c. What is the DFT of $X[m]$?

d. Suppose you are given another sequence $w[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$. Let $C[m]$ denote the cyclic convolution of $x[n]$ and $w[n]$. Compute the values of $C[1]$ and $C[3]$.

Problem 9{15 Points}

In (a)-(c), $x[n]$ denotes the input of a system and $y[n]$ denotes its output.

(a) $y[n] = x[2n]$
Is the system causal? (Yes/No)
Justify your answer:

(b) $y[n] = n^2 x[2n]$
Is the system time-invariant? (Yes/No)
Justify your answer:

(c) $y[n] = x^3[2n]$
Is the system linear? (Yes/No)
Justify your answer: